Proceedings of
2003 STLE/ASME Joint International Tribology Conference
Ponte Vedra Beach, Florida USA, October 26-29, 2003

2003-TRIB-265

FRICTION MODELING OF A FREE-SPINNING BICYCLE WHEEL

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ABSTRACT

An experiment is conducted in which a free-standing bicycle wheel is given an initial angular speed and then allowed to slow down to rest. Measurements of the wheel speed, using a magnetic sensor, during the decay are compared to predictions from a model accounting for a combination of viscous and dry friction at the wheel bearing. The time history data indicate two dynamic regimes: *(i)* a higher speed phase corresponding to the first part of the motion for which a simple viscous friction model applies, and *(ii)* a slower speed phase corresponding to low speed to stop behavior for which a model involving both viscous and dry friction is proposed. A method is presented for finding the viscous and dry friction coefficients of the two phases.

1 INTRODUCTION

Anyone who has spun a bicycle wheel, with the wheel off the ground, can attest to the fact that the slowly decaying behavior has an almost mesmerizing effect. Wheel inertia keeps the wheel rotating for a long time, making the reduction in speed almost imperceptibly detectable. But, over time, the wheel speed decreases and eventually the wheel comes to rest. If the wheel is well balanced and/or if there is enough bearing friction, the wheel smoothly stops rotating. However, if the wheel is not well balanced and there is minor bearing friction, the wheel may exhibit an oscillatory tendency near the end of the motion. Prior to coming to rest, the wheel may rotate back and forth, oscillating like a pendulum, after losing most but not all of its rotational energy.

The rotational motion of a bicycle wheel about its spin axis, where the wheel has been imparted with an initial angular velocity, is an interesting and educational subject. We have introduced an experiment on bicycle wheel speed decay in course “MEEN 120: Mechanical Measurements and Instrumentation” at Marquette University. The laboratory experience requires students to study the dynamics of a free-spinning wheel using a non-contact magnetic speed sensor and a data acquisition system. The impetus for the experiment was a similar experiment introduced at the University of Washington (Seattle, WA) by Professors Joseph Garbini and William Murray, and reported in [1].

The physical system of a freely rotating bicycle wheel can be represented by a relatively simple model involving a rotational inertia and a rotational viscous damper. This linear model can be solved and predicts an exponentially decaying speed starting from the initial speed. The model can also be used to introduce the concept of time constant, which can be found experimentally. (It is the negative slope of the plotted line in a graph of the logarithm of angular speed versus time.) With the time constant, and with knowledge of, or an estimate of, the wheel rotational inertia, the value of the viscous damping coefficient can be determined.

In theory, this first-order model predicts that the wheel continues to rotate ad infinitum. A more advanced model can be posited including both viscous damping and dry friction, where the latter is introduced to ensure that the wheel stops in finite time. One challenge of this model is that there are two friction parameters that must be determined.

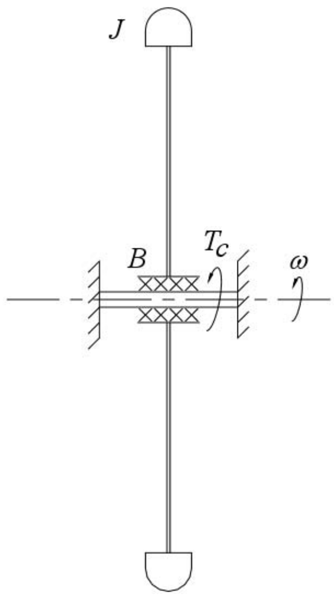

Figure 1. Lumped parameter model of bicycle wheel.

1.1 Scope

In this paper, the rotational speed decay of a free-standing bicycle wheel is investigated and an appropriate dynamic model that accounts for frictional losses is developed. The model presented includes both viscous damping and Coulombic dry friction effects. The results of simulation and experimental studies are presented.

2 DYNAMIC MODEL

Consider a bicycle wheel in a vertical plane spinning freely about its axis. It is assumed that the wheel is balanced (with its mass center at the wheel's geometric center). A dynamic model is proposed involving wheel inertia and two retarding effects of viscous and dry friction at the wheel bearing, as depicted schematically in Figure 1. The viscous friction torque is linearly related to the angular velocity, and the dry friction is constant (according to Coloumb's law).

The equation of motion is:

$$J\dot{\omega} + B\omega + T_c = 0, \tag{1}$$

with initial condition

$$\omega(0) = \omega_0 \tag{2}$$

where J is the moment inertia of the wheel, B is the viscous damping coefficient and T_c is the friction torque (assumed constant). Solving gives

$$\omega(t) = \left(\omega_0 + \frac{T_c}{B}\right) e^{-\frac{t}{\tau}} - \frac{T_c}{B} \tag{3}$$

where the time constant, τ, is

$$\tau = \frac{J}{B}. \tag{4}$$

The model has a physical inconsistency in that Eq. (3) is not a solution for all $t \in [0, \infty]$. The predicted terminal speed is $\omega(t \to \infty) = -T_c/B$, which is not physically consistent for a nonzero T_c.

It is possible to determine the time t^* when the wheel comes to rest. By setting $\omega(t^*) = 0$ in (3),

$$\left(\omega_0 + \frac{T_c}{B}\right) e^{-\frac{t^*}{\tau}} - \frac{T_c}{B} = 0, \tag{5}$$

and solving yields:

$$t^* = -\tau \ln\left(\frac{T_c}{B\omega_0 + T_c}\right).$$

Introducing

$$\lambda = \frac{T_c}{B}, \tag{6}$$

t^* can be written as:

$$t^* = -\tau \ln\left(\frac{\lambda}{\omega_0 + \lambda}\right). \tag{7}$$

Equation (3) applies only for $t \le t^*$. For $t \ge t^*$, $T_c = 0$ and $\omega(t) \equiv 0$. Thus, the motion of the wheel can be described by:

$$\omega(t) = \begin{cases} (\omega_0 + \lambda) e^{-\frac{t}{\tau}} - \lambda & \text{...if } t \le t^* \\ 0 & \text{...if } t > t^* \end{cases} \tag{8}$$

The model is characterized by two parameters, τ and λ. τ indicates the influence of viscous friction whereas λ is the ratio of dry friction torque to viscous friction damping. From knowledge of these two parameters and a known inertia J, the viscous damping coefficient and the dry friction torque can be obtained.

For $\omega_0 \gg \lambda$, then Eq. (3) can be approximated by:

$$\omega(t) \approx \omega_0 e^{-\frac{t}{\tau}} \tag{9}$$

which is suitable for an appropriate range of $t < t^*$. However, for t less than but closer to t^*, the term λ cannot be neglected.

2.1 Two Phase Model

Based on the above, a two phase model is proposed. At higher speeds, dry friction (indicated by λ) is neglected, and the angular speed of the wheel is written in the form of Eq. (9):

$$\omega(t) = \omega_0 e^{-\frac{t}{\tau}}. \tag{10}$$

At lower speeds, dry friction must be taken into account, and the angular speed of the wheel has the form of Eq. (3):

$$\omega(t) = (\omega_0 + \lambda)e^{-\frac{t}{\tau}} - \lambda. \tag{11}$$

Let t_c is the "critical" (or "boundary") time switching between the two phases. Noting that at $t = t_c$ the angular speed must be continuous, the speed can be written:

$$\omega(t) = \begin{cases} \omega_0 e^{-\frac{t}{\tau}} & \text{...for } t \le t_c \\ \left(\omega_0 e^{-\frac{t_c}{\tau}} + \lambda\right) e^{-\frac{t-t_c}{\tau}} - \lambda & \text{...for } t_c \le t \le t^* \\ 0 & \text{...for } t > t^* \end{cases} \tag{12}$$

Another possible model accounts for the possibility that τ in Eqs. (10) and (11) may not necessarily be the same. To distinguish between the two, they are denoted by τ_1 and τ_2 corresponding to the time constant in the higher speed and lower speed phases, respectively. Thus, the angular speed can be written:

$$\omega(t) = \begin{cases} \omega_0 e^{-\frac{t}{\tau_1}} & \text{...for } t \le t_c \\ \left(\omega_0 e^{-\frac{t_c}{\tau_1}} + \lambda\right) e^{-\frac{t-t_c}{\tau_2}} - \lambda & \text{...for } t_c \le t \le t^* \\ 0 & \text{...for } t > t^* \end{cases} \tag{13}$$

3 METHOD

3.1 Optimization to Determine Parameters

In order to determine the parameters, either τ, λ and t_c for Eq. (12) or τ_1, τ_2, λ and t_c for Eq. (13), an optimization approach is suggested. The proposed objective function, F, is:

$$F = \sum_{i=1}^{n} [\omega(t_i) - \omega_i]^2 \tag{14}$$

where t_i and ω_i correspond to the time and angular speed, respectively, at the i-th data point obtained from an experiment, n is the total number of data points, and $\omega(t)$ is defined by Eq. (12) or (13).

The optimization problem is to minimize F subject to $\tau_i > 0$, $\lambda > 0$ and $0 < t_c < t^*$, where t^* is given by Eq. (7). The initial point for the optimization can be chosen arbitrarily for $\tau_{i0} > 0$, $\lambda_0 > 0$ and $t_{c0} < \tau_{i0} \ln(\frac{\lambda_0}{\omega_0 + \lambda_0})$. The first speed in the time history data is the initial speed ω_0 for both models (12) and (13). The optimization stops when a set of parameters (τ_i, λ, t_c) that minimize F is found. The optimization was performed using MATLAB and its Optimization Toolbox.

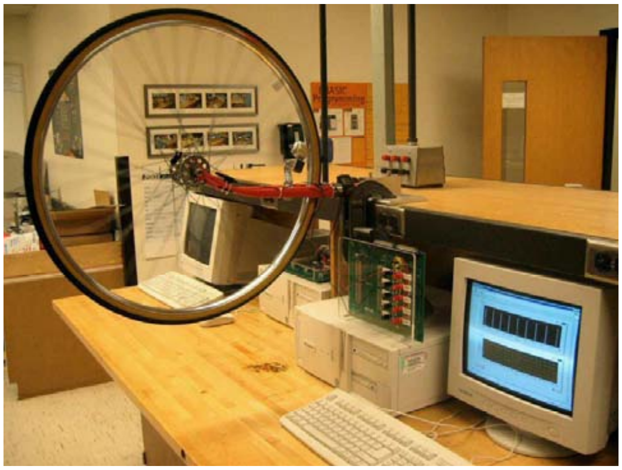

Figure 2. Experimental setup showing bicycle wheel on front fork clamped to laboratory bench and data acquisition system.

3.2 Experimental Setup

The front fork of a bicycle was clamped to a laboratory table, as shown in Figure 2. (A 27 inch wheel on a front fork assembly from a street bicycle was used in the test.) A non-contact magnetic speed sensor (CAT EYE Velo 2) was mounted to one leg of the fork. It provides a single voltage pulse as a small magnet mounted to a spoke passes, giving one pulse per revolution during the experiment. Data acquisition was accomplished using LabVIEW. The wheel was spun manually, and speed versus time data were collected.

4 RESULTS

Figure 3 compares the angular speed versus time data for an experiment with an initial angular velocity of $\omega_0 = 15.140$ rad/s to the predicted speed from the viscous model of Eq. (10). In this

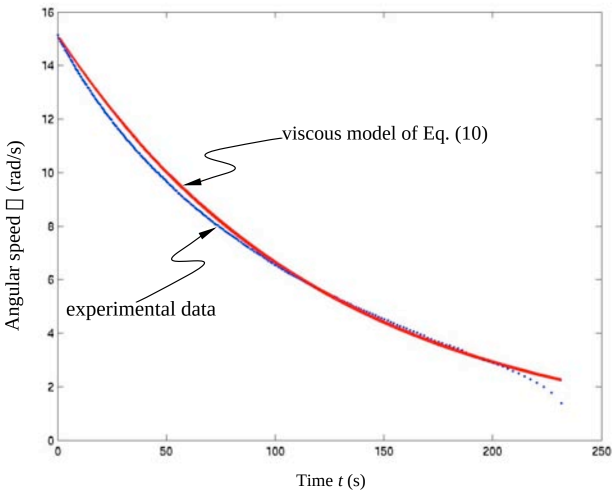

Figure 3. Angular speed ω vs. time t. Experimental data shown in blue. Viscous model of Eq. (10) shown in red.

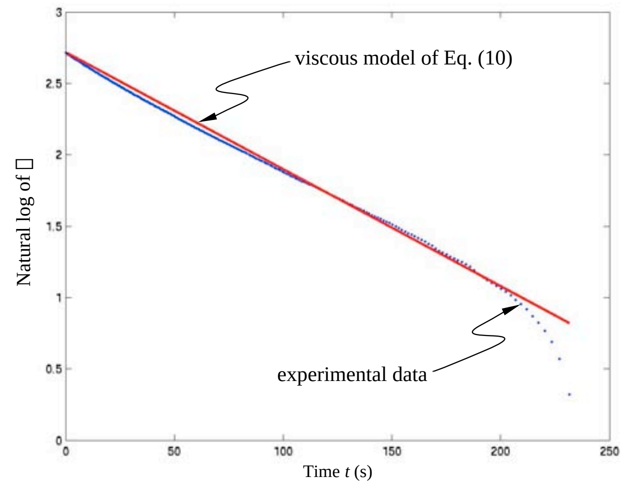

Figure 4. Natural logarithm of ω vs. time t. Experimental data shown in blue. Viscous model of Eq. (10) shown in red.

experiment, the wheel speed decayed for approximately 6 minutes. The wheel used was not perfectly balanced, and oscillated slightly prior to coming to rest. The data presented in Figure 3 is for the wheel angular speed prior to this oscillation.

Figure 4 presents the same angular speed history data in terms of the natural logarithm of the speed versus time. Also shown in the figure is a comparison to the viscous model based on the best-fit line over the time period $0 < t < 150\,\text{s}$. The τ corresponding to this best-fit line was found to be 121 s, and this value was used to determine the viscous model prediction in Figure 3.

The comparison of the experimental data to the viscous model of Eq. (10) with this value of τ indicates a close match for all but the slowest speeds. The viscous model is a very reasonable approximation for the higher speeds. Figure 4 illustrates the excellent fit of the data to a straight line, with the slope of $-1/\tau$, corresponding to the log of the function $\omega(t)$ of Eq. (10). The deviation from the straight line at the slowest speeds suggests that the viscous model alone does not capture the full behavior. It is suggested below that the influence of λ cannot be neglected as the wheel comes to rest.

For the experimental data illustrated in Figures 3 and 4, the parameters resulting from the optimization were obtained as follows. For the two phase model with same viscous coefficient, i.e., the model described by Eq. (12), the parameters from the optimization are: $\tau = 117.85\,\text{s}$, $\lambda = 1.515\,\text{rad/s}$, and $t_c = 198.00\,\text{s}$. For the two phase model with different viscous coefficients, i.e., the model described by Eq. (13), the parameters from the optimization are: $\tau_1 = 106.04\,\text{s}$, $\tau_2 = 159.37\,\text{s}$, $\lambda = 1.480\,\text{rad/s}$, and $t_c = 42.486\,\text{s}$.

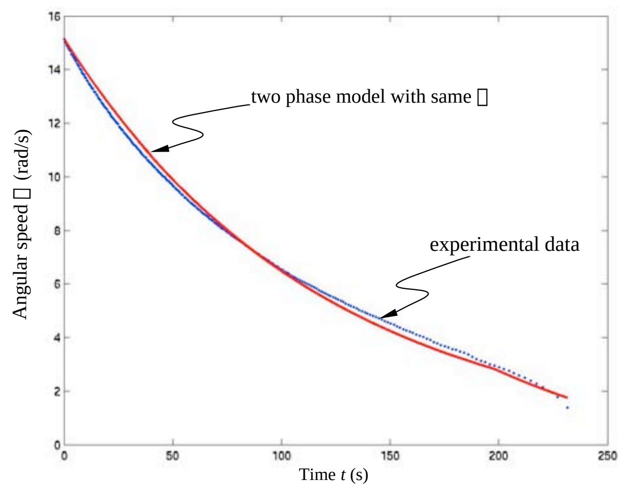

Figure 5. Angular speed time history for two phase model with same viscous coefficient. Experimental data shown in blue. Model of Eq. (12) with optimized parameters shown in red.

Figures 5 and 6 show the $\omega(t)$ curves with the above parameters for the two models, respectively, where the comparisons to the experimental data are also illustrated. The results suggest a close match in both cases, especially for the model with different viscous coefficients. It is possible to quantify how well the different models match the data by calculating the value of the objec-

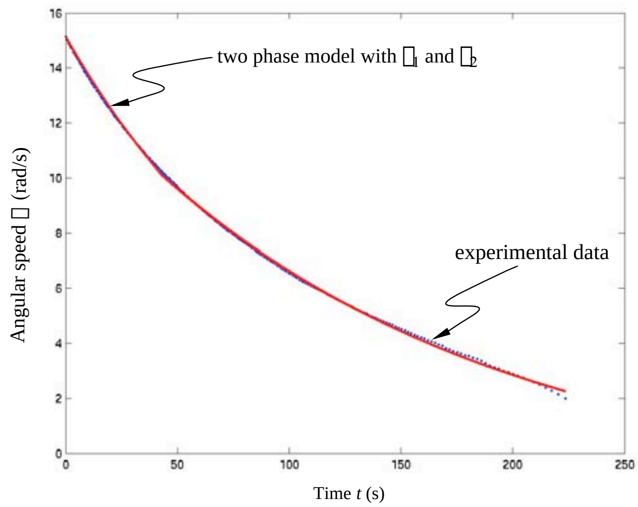

Figure 6. Angular speed time history for two phase model with different viscous coefficients. Experimental data shown in blue. Model of Eq. (13) with optimized parameters shown in red.

tive function of Eq. (14). For the one phase viscous model of Eq. (10), the value of the objective function is $F = 18.44$ (rad/s)^2. In comparison, for the two phase models the results are $F = 11.83$ (rad/s)^2 for the model of Eq. (12) and $F = 1.94$ (rad/s)^2 for the model of Eq. (13). The closest match is for the more advanced two phase model with different time constants.

5 CLOSING

This paper presents a multi-parameter model for bearing friction accounting for viscous damping and Coulomb friction that is useful in investigating the dynamics during free-spin slow down of a bicycle wheel. Measurements of the wheel speed during the decay are compared with predictions of a model accounting for a combination of viscous and dry friction at the wheel bearing. The time history data indicate two dynamic regimes, i.e., a higher speed regime corresponding to the first part of the motion for which a simple viscous friction model applies, and a slower speed regime corresponding to low speed to stop behavior for which a model involving both viscous and dry friction seems appropriate. The experiment fosters creative thinking about the modelling of a basic dynamics problem. It is well suited for undergraduate courses in "dynamics" and in "measurement and instrumentation."

ACKNOWLEDGEMENT

The authors are grateful to Mr. Norman Kirchner for setting up the bicycle wheel experiment and implementing the data acquisition system. The authors also wish to thank Messrs. Tim Dake, Casey Loughrin, and Tom Silman for their input and help with the experiment. The authors are indebted to the Kern Family Foundation for their support of the Discovery Learning Center, where the initial tests were conducted. We also gratefully acknowledge the College of Engineering Associate Dean G.E.O. Widera and the Department of Mechanical and Industrial Engineering Chair K. Kim for unwavering support of the Mechanical Measurements, Instrumentation, and Mechatronics Laboratory.

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