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PREDICTING SEGMENT TRAJECTORIES OF A LOCOMOTION MODEL BY A SUBOPTIMAL CONTROL ALGORITHM

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ABSTRACT

This research focuses on an optimal control approach to simulate limb segment motions of a planar mechanical walking model. In this approach the optimal segment motions are a function of a performance index (such as mechanical energy) which is minimized and physically-based system constraints which must be satisfied. A Fourier-based approximation technique is applied to convert the optimal control problem into a nonlinear programming problem which can be solved using well-developed optimization algorithms. The set of nonlinear algebraic equations subject to constraints is solved for the suboptimal histories of joint angles, velocities, accelerations, and torques. By investigating different performance indices and comparing the resulting motion histories with human walking data, the approach can be used to study strategies that humans use in selecting dynamic patterns of limb motions during locomotion.

INTRODUCTION

Three different approaches used to study bipedal locomotion are the direct dynamic, the inverse dynamic, and the optimal control methods. The direct dynamic approach views generalized forces (forces and moments) as system inputs. By solving the equations of motion of a bipedal locomotion model, the time history of generalized coordinates (joint angles) and their derivatives (velocities and accelerations) can be obtained. There are obvious difficulties with measuring the generalized forces, and in using this method the predicted trajectories typically contradict those of the real system.

The inverse dynamic approach uses the time history of the generalized coordinates and their derivatives as inputs. From the equations of motion, the corresponding generalized forces can be obtained. This approach is computationally efficient and has been widely used in bipedal locomotion research. The major disadvantage of the inverse dynamic approach is its sensitivity to measurement noise which is problematic when differentiating measured joint displacements to obtain joint velocities and accelerations [1].

The optimal control approach assumes a "principal of optimality" such as minimum energy expenditure during human locomotion as suggested by oxygen consumption data [2]. Once a performance index and system constraints are identified, the optimal trajectory of a model of known parameters can be determined . One difficulty with the optimal control approach is associated with selecting (and expressing analytically) the performance index. A second difficulty is related to computer implementation. Standard optimal control algorithms are typically very sensitive to numerical errors and/or require a very large amount of computer memory. As a result, previous optimal control approaches applied to bipedal locomotion have been limited to single leg models [3,4].

This paper proposes an optimal control algorithm for predicting segment trajectories during normal locomotion of a mechanical model. Many of the implementation difficulties encountered in applying optimal control theory to high-order, nonlinear models are avoided by approximating the generalized coordinates and their rates by finite terms of Fourier series.

METHODOLOGY

Bipedal Locomotion Model.

The human musculoskeletal system is represented by a two-dimensional, five-link, rigid-body model, as shown in Figure 1, driven by ideal torque actuators at the articulations. The head, arms, and torso (HAT) are modeled by a single link. Each leg is modeled by two links, representing the thigh and the shank, with a massless foot attached rigidly to the shank. The dimensions and mass inertial properties of all links are assumed known.

Normal bipedal locomotion is considered to be a sequence of walking cycles, with each cycle consisting of two steps. Each step includes two "phases" - - a single and double stance phase - - defined according to the number of legs in contact with the ground. Here, the single stance phase of step k consists of time interval $t_{k0} \leq t < t_{k1}$; the double stance phase consists of time interval $t_{k1} \leq t \leq t_{kf}$. The initial time t_{k0} corresponds to "toe-off" of one leg, while the onset of double stance, time t_{k1}, corresponds to "heel-strike" of the same leg. The final time t_{kf} coincides with "toe-off" of the other leg. Thus, the time to complete one walking step k is $t_{kf} - t_{k0}$.

Performance Index.

The process of optimizing the motion histories involves minimizing a scalar performance index (or objective function), J,

$$J = \int_{t_{k0}}^{t_{kf}} g\ (\underline{T}(t)\ ,\ \underline{\theta}(t)\ ,\ \underline{\dot{\theta}}(t)\ ,\ \underline{\ddot{\theta}}(t)\ ,\ t)\ dt \qquad (1)$$

where g represents a general function, $\underline{T}$ is the vector of joint (and ground pivoting) torques, $\underline{\theta}(t)$, $\underline{\dot{\theta}}(t)$, and $\underline{\ddot{\theta}}(t)$ are the vectors of joint angular displacements, velocities, and accelerations, respectively, and t is time. For example, $g = \underline{T}^{*}(t)\underline{\dot{\theta}}(t)$, where the superscript * represents transpose, if the performance index is the mechanical energy associated with step k. Since the behavior of the bipedal locomotion model is governed by the dynamic equations of motion for which $\underline{T}(\underline{\theta}(t)\ ,\ \underline{\dot{\theta}}(t)\ ,\ \underline{\ddot{\theta}}(t))$, the integrand of equation (1) can be written as a function of joint angles, velocities, accelerations, and time.

Kinematic Constraints.

Kinematic constraints can be divided into those that are necessary to represent normal human locomotion (#1 to #4 listed below), and those that are not actually necessary but are assumptions imposed to simplify the problem (#5 and #6).

1. $\theta_2(t) \geq \theta_1(t)\ ,\ \theta_4(t) \geq \theta_5(t)$ — for $t_{k0} \leq t \leq t_{kf}$

   This constraint guarantees that the angular displacement of each thigh is not less than the angular displacement of each shank.

2. $y_1(t) > 0$ — for $t_{k0} \leq t \leq t_{k1}$

   Displacement y_1 represents the vertical clearance of the "toe" of the swing leg. Thus, this constraint ensures that the "toe" of the swing leg clears the ground during the single stance phase.

3. $y_2(t) = 0\ ,\ \dot{y}_2(t) = 0\ ,\ \ddot{y}_2(t) = 0$ — for $t_{k1} \leq t \leq t_{kf}$

Displacement y_2 represents the vertical clearance of the "ankle" of the leg that has just entered stance. Thus, this constraint guarantees that during the double stance phase, the foot is in contact with the ground.

4. $x_2(t)$ = const , $\dot{x}_2(t) = 0$, $\ddot{x}_2(t) = 0$ .... for $t_{k1} \le t \le t_{kf}$

   Displacement x_2 is the horizontal distance between the "toe" of one leg and the "ankle" of the other leg. This constraint ensures that the "pivoting" on the centers of pressure of both feet occurs during the double stance phase.

5. $\theta_3(t) = 90°$ ...... for $t_{k0} \le t \le t_{kf}$

   This simplifying assumption, that the HAT link is fixed vertically, is based on the fact that the angular motion of the HAT is normally much smaller than the motions of the lower limbs.

6. $\dot{x}_3(t) = V_x$ ...... for $t_{k0} \le t \le t_{kf}$

   Velocity $\dot{x}_3(t)$ represents the absolute velocity of the hip. Thus, this constraint dictates that the model moves with a constant forward speed, V_x, which is reasonable during normal steady walking.

Trajectory Generation Algorithm.

The following trajectory generation algorithm (for step k) converts the optimal control problem into a nonlinear programming problem using a Fourier-based approximation technique. We start by letting the nth joint angle $\theta_n(t)$ be expressed as

$$\theta_n(t) = \alpha_n(t) + \beta_n(t) \tag{2}$$

where $\alpha_n(t)$ is a fifth-order auxiliary polynomial

$$\alpha_n(t) = d_{n0} + d_{n1}t + d_{n2}t^2 + d_{n3}t^3 + d_{n4}t^4 + d_{n5}t^5 \tag{3}$$

and $\beta_n(t)$ is a Fourier-type series

$$\beta_n(t) = \sum_{m=1}^{M} \left\{ a_{mn} \cos\left[\frac{2m\pi\ (t - t_{k0})}{t_{kf} - t_{k0}} \right] + b_{mn} \sin\left[\frac{2m\pi\ (t - t_{k0})}{t_{kf} - t_{k0}} \right] \right\} \tag{4}$$

Equations (2)-(4) apply for n = 1, . . ., N where N is the number of joint angles (or generalized coordinates) and M is the number of terms in the Fourier-type series approximation. For finite M, the formulation represents a truly optimal control approach; for finite M, the method provides a suboptimal solution. The six coefficients of the auxiliary polynomial can be selected to satisfy the initial and terminal conditions on $\theta(t_{k0})$, $\theta(t_{kf})$, $\dot{\theta}(t_{k0})$, $\dot{\theta}(t_{kf})$, $\ddot{\theta}(t_{k0})$, $\ddot{\theta}(t_{kf})$.

The process of optimizing the motion histories involves minimizing the performance index J ($\underline{A}$, $\underline{B}$, $\underline{\theta}(t_{kf})$, $\underline{\dot{\theta}}(t_{kf})$, $\underline{\ddot{\theta}}(t_{kf})$, t_{kf}) where $\underline{A}$ and $\underline{B}$ represent M x N matrices with elements a_{mn} and b_{mn} , respectively, without violating the constraints. (We can assume $\underline{\theta}(t_{k0})$, $\underline{\dot{\theta}}(t_{k0})$, and $\underline{\ddot{\theta}}(t_{k0})$ are known initial conditions or can guess their values and let the model continue until steady conditions are achieved.) Since J is not a function of t, we have converted a time dependent optimal control problem into a standard nonlinear programming problem. Here, the performance index is numerically integrated using a routine method such as Simpson's composite integral method. In solving the nonlinear programming problem, we can draw upon the wealth of existing nonlinear optimization algorithms such as direct search and penalty function methods. The resulting (sub)optimal values of $\underline{A}$, $\underline{B}$, $\underline{\theta}(t_{kf})$, $\underline{\dot{\theta}}(t_{kf})$, $\underline{\ddot{\theta}}(t_{kf})$, and t_{kf} uniquely determine the (sub)optimal joint variables (angular displacements, rates, and torques) which can be calculated by straight-forward algebra.

DISCUSSION

The proposed approach offers efficient utilization of computer memory. The time history of each of the joint angular displacements is approximated by a single function and the optimality is found by adjusting a relatively small number of parameters for each of these functions. As such, computational and dimensionality problems typical of the traditional optimal control implementation are avoided.

The effectiveness of the suboptimal control approach has been tested by a series of computer simulations. From the simulation results, we have found that satisfactory results can usually be achieved by using two or three terms of the Fourier-type expansion functions.

CONCLUSIONS

This paper proposes an optimal control approach for studying bipedal locomotion. A Fourier-based approximation scheme for obtaining the suboptimal trajectories of a bipedal model has been formulated. This scheme offers a new means to handle high order, nonlinear bipedal models, which previous optimal control studies have not investigated. The algorithm requires specification of model parameters, a performance index, system constraints, and no additional experimental data. Currently, we are simulating the optimal segment trajectories of the given model and results will be reported in the future.

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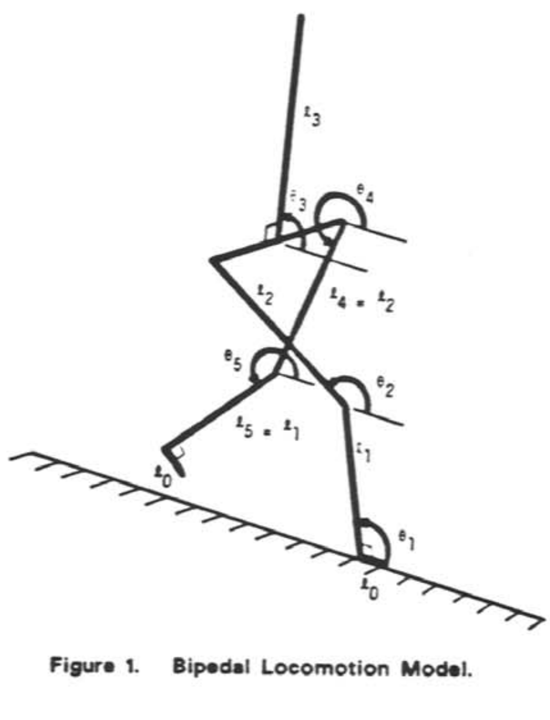

Figure 1. Bipedal Locomotion Model.