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A THEORETICAL APPROACH FOR OPTIMAL MOTION GENERATION OF A BIPEDAL LOCOMOTION MODEL

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INTRODUCTION

Human walking is a very complicated dynamic behavior. There is strong interest in the medical and engineering communities to understand the dynamic patterns of limb segment motions during locomotion to help identify normal and abnormal gaits, to help people with gait pathologies (*e.g.*, by rehabilitation or corrective surgery), and to build better orthotic and prosthetic devices.

In this research, we develop a methodology for simulation of optimal motions of a planar mechanical walking model. The approach involves minimization of a performance index subject to system constraints and is formulated as a problem in calculus of variations using the Raleigh-Ritz technique. The result is a set of nonlinear algebraic equations which is solved using the simplex search direct optimization algorithm for optimal histories of joint angles, rates, and torques. The method is designed to help uncover strategies that play a role in the coordinated management of limb motions during human locomotion.

BACKGROUND

Studies [1] suggest that during locomotion humans minimize oxygen consumption. As a result, it has been a classical argument that the mechanism for selection of limb segment motions during gait is predicated upon a criterion of minimum energy consumption.

There are two principal approaches for studying human locomotion. The typical approach, called the inverse dynamic approach, is to collect human movement data and apply classical principles of mechanics to a linkage model to predict the resultant joint forces and moments that "caused" the movement. In obtaining the required kinematic data, difficulties with numerical differentiation of displacements are often encountered. The second, less popular approach is a direct dynamic approach in which joint initial conditions and joint loads are measured directly and used to calculate joint angles and velocities of a linkage model by integration of the governing equations of motion. To simulate a typical gait pattern without violating physically-based system constraints (such as ground clearance during swing), trial and error perturbations of the inout torques are usually required due to imperfect models.

The inverse and direct dynamic approaches rely heavily on experimental data, provide limited insight into optimality, and are not well suited to incorporate system constraints. This paper describes an alternative scheme to determine the optimal limb segment motions (in terms of joint angles, velocities, and accelerations) of a bipedal model during each "step" of straight-line, level locomotion.

METHODOLOGY

Model. A two-dimensional, articulated, five-link, rigid-body model is used to represent the human body. As shown in Figure 1, one link represents the lumped upper body which includes the head, arms, and torso, two links represent the upper legs, and two links represent the lower legs. (In the figure, the link connecting the "hips" is rigidly attached at right angles to the upper body link.) There are no links representing the feet nor elements representing stiffness and damping properties. The mass inertial properties of all links are assumed known. The links representing the two legs have identical dimensions and inertial properties. The stance leg pivots at its contact with the ground. The linkage model is driven by pure torque actuators at the joints representing the effect of muscles. For the stance leg, a torque is applied at the supporting joint representing the ankle/foot.

The behavior of the model is governed by the dynamic equations of motion given by:

$$\underline{T} = \underline{T}(\underline{\theta}(t), \dot{\underline{\theta}}(t), \ddot{\underline{\theta}}(t)) \tag{1}$$

where $\underline{T}$ is the vector of joint torques, $\underline{\theta}(t)$, $\dot{\underline{\theta}}(t)$, and $\ddot{\underline{\theta}}(t)$ are the vectors of joint angles, velocities, and accelerations, respectively, and t is time.

Assumptions of Bipedal Locomotion. Walking is considered to be a sequence of steps, with two steps constituting one gait cycle. During each step one leg is in a stance phase while the other leg is in a swing phase. The legs alternate between stance and swing phases, with "toe-off" of the stance leg corresponding with "heel-strike" of the swing leg both occurring at an instant of double stance. At this instant, the initial time of step k, t_{ko}, coincides with the final time of step k−1, $t_{(k-1)f}$. The step time is $\Delta t_k = t_{kf} - t_{ko}$.

Performance Index. The process of optimizing the motion histories involves minimizing a scalar performance index (or objective function), J,

$$J = \int_{t_{ko}}^{t_{kf}} g(\underline{T}(t), \underline{\theta}(t), \dot{\underline{\theta}}(t), \ddot{\underline{\theta}}(t), t)\,dt \tag{2}$$

where g is an arbitrary function. For example, $g = \underline{T}^{*}(t)\,\dot{\underline{\theta}}(t)$, where * represents transpose, if the performance index is the mechanical energy associated with the motion in step k.

System Constraints. To achieve bipedal locomotion, the mechanical model must satisfy physically-based system constraints, five of which are identified below:

(i) Desired Forward Velocity. The horizontal velocity of a point on the upper body link is constrained to be a known function of time. Here, we assume the uppermost point of link 3, representing the head, is a known constant forward velocity, V_x.

$$V_x = -\sum_{i=1}^{3} \ell_i \dot{\theta}_i \sin\theta_i \ , \ldots\ldots \ t \in [t_{ko}, t_{kf}] \tag{3}$$

where ℓ_i represents the length of link i.

(ii) Desired Upper Body Attitude. The attitude angle of the upper body link is constrained to be a known function of time. Here, we assume the link is perfectly vertical during each step.

$$\theta_3(t) = \frac{\pi}{2}, \ldots\ldots\ldots\ldots \ t \in [t_{ko}, t_{kf}] \tag{4}$$

(iii) Swing Leg Clearance. The foot of the swing leg must not go below the ground.

$$\sum_{i=1, i\neq 3}^{5} \ell_i \sin\theta_i \geq 0 \ , \ldots\ldots \ t \in [t_{ko}, t_{kf}] \tag{5}$$

(iv) Swing Leg Vertical Displacement at Heel Strike. The foot of the swing leg must have zero vertical displacement at heel strike.

$$\sum_{i=1, i\neq 3}^{5} \ell_i \sin\theta_i = 0 \ , \ldots\ldots \ t = t_{kf} \tag{6}$$

(v) Hip Forward Velocity at Heel Strike. The horizontal velocity of the hip must be the same (*i.e.*, continuous) for both legs at heel strike.

$$\sum_{i=1, i\neq 3}^{5} \ell_i \dot{\theta}_i \sin\theta_i = 0 \ , \ldots\ldots \ t = t_{kf} \tag{7}$$

Raleigh-Ritz Approximation. The Raleigh-Ritz scheme [2] is employed to approximate each joint angle as follows:

$$\theta_i(t) \cong \phi_{i0}(t) + \sum_{j=1}^{m} C_{ij}\,\phi_j(t)\,, \qquad i = 1, \ldots, n \tag{8}$$

where m equals the number of terms in the series and n equals the number of joint angles (*i.e.*, n = 5). In eq(8) the C_{ij} are unknown weighting coefficients, the $\phi_j(t)$ are approximating shape functions assumed here to be

$$\phi_j(t) = 1 - \cos\frac{2j\pi(t-t_{ko})}{\Delta t_k} \tag{9}$$

such that $\phi_j(t_{ko}) = \phi_j(t_{kf}) = 0$, and $\phi_{i0}(t)$ is a function chosen to satisfy initial and final conditions at each step. Here

$$\phi_{i0}(t) = \sum_{j=0}^{r} b_{ij}\,t^j \tag{10}$$

where r = 4 for θ_i and $\dot{\theta}_i$ specified at $t = t_{ko}$ and $t = t_{kf}$. The coefficients b_{ij} are determined from joint angles and velocities at $t = t_{ko}$ (assumed known data for k = 1 and obtained from the final conditions of the last step for k > 1) and from system constraints (6) and (7). In summary, the vector of joint angles can be approximated as:

$$\underline{\theta}(t) \cong \underline{f}_1(t, \underline{C}, \Delta t_k) \tag{11}$$

where $\underline{f}_1$ represents a vector of functions and $\underline{C}$ is the n x m matrix of C_{ij}. It follows that the joint velocity and acceleration vectors are $\dot{\underline{\theta}}(t) \cong \underline{f}_2(t, \underline{C}, \Delta t_k)$ and $\ddot{\underline{\theta}}(t) \cong \underline{f}_3(t, \underline{C}, \Delta t_k)$, respectively, where $\underline{f}_2$ and $\underline{f}_3$ represent vectors of functions.

Optimization Strategy. With the Raleigh-Ritz approximation, the performance index can be rewritten as a system of nonlinear equations:

$$J = J(\underline{C}, \Delta t_k) \tag{12}$$

The problem that remains is a numerical one, *i.e.*, we need to find $\underline{C}$ and Δt_k that minimize J and satisfy system constraints (3), (4), and (5) not used above . We use the simplex search direct optimization algorithm [3] to solve this problem. With this approach divergence is almost impossible and there is no need to evaluate derivatives. Equality constraints (3) and (4) are included by reducing the system order and inequality constraint (5) is included by introducing a penalty function. Once we solve for $\underline{C}$ and Δt_k, we can reconstruct the optimal joint vectors $\underline{\theta}(t)$, $\dot{\underline{\theta}}(t)$, $\ddot{\underline{\theta}}(t)$, and $\underline{T}(t)$.

DISCUSSION

A technique has been developed to generate the optimal motion histories of a planar, articulated linkage model simulating bipedal locomotion. (The algorithm can also be used to generate optimal motion programs for bipedal walking robots.) The predicted joint motions can be compared with clinically measured joint data. Thus, the method enables one to investigate gait as an "optimal solution" with respect to energy, stability, *etc.* By testing and adjusting different performance indices until the predicted and measured data match, a plausible model of the performance index that humans minimize in normal bipedal progression can be identified.

It is hypothesized that in abnormal walking, the model of the performance index is the same as the model identified for normal locomotion but the system constraints are different. For instance, the maximum torque at one joint might be limited due to injury. In general, the formulation is flexible in that it can incorporate a variety of constraints, including anthropomorphic limits on joint angles and torques, patient-specific constraints, ground reaction data, and non-level walking constraints.

Finally, the method can be used to study different mechanical models of human locomotion, such as models with additional links (for the feet, arms, *etc.*) and with elements representing stiffness and damping properties at the joints.

SUMMARY

We offer a variational approach that deals directly with optimality and system constraints to study bipedal locomotion. In addition to predicting limb segment motions during walking, the method represents a useful analytical tool to explore dynamic characteristics such as joint torque histories, ground reaction loads, and center of gravity behavior. It is hoped that the method will contribute to the understanding of limb segment motions during bipedal locomotion.

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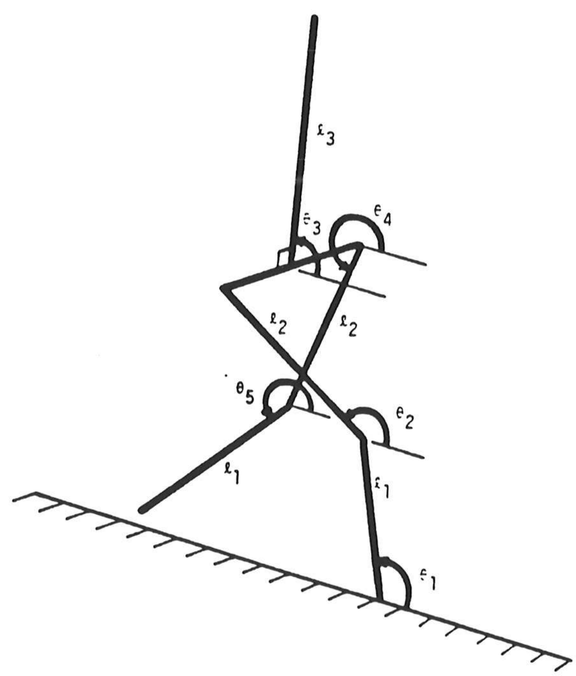

Figure 1. Bipedal Locomotion Model.