A SIMPLE DYNAMICS EXPERIMENT BASED ON ACOUSTIC EMISSION

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This paper describes a simple experiment, well suited for an undergraduate course in mechanical measurements and/or dynamics, in which physical information is extracted from an acoustic emission signature. In this experiment, a ping-pong ball is dropped onto a hard table surface. A microphone and computer sound system are used to record the audio signal resulting from the ball-table impacts. The times between successive bounces, viewed as "flight times," are used to determine the height of the initial drop and the coefficient of restitution of the impact.

1. INTRODUCTION

Acoustic emission is an example of an indirect measurement technique in which clues to the behavior of a physical system are obtained via sound signatures. Sounds waves are often "incidental" manifestations of dynamic behavior, and provide the opportunity to learn much about the physical world. This paper describes an experiment in which physical information is extracted from an acoustic emission signature. It is simple to conduct and well suited for undergraduate engineering courses such as "mechanical measurements" and "dynamics".

In this experiment, a ping-pong ball is dropped from rest onto a hard table surface, and the acoustic signatures of the ball-table impacts are recorded using a microphone and computer sound system. The times between bounces, or "flight times," are found from the temporal history of the bounce sounds of the successive impacts. These flight times are used to calculate the height of the initial drop and the coefficient of restitution of the impact. The experiment is straightforward and affords students the opportunity to compare their results with established theoretical concepts.

1.1 Background: Table Tennis

The official rules of table tennis[1] specify the characteristics and properties of a ping-pong ball: "The ball shall be spherical, with a diameter of 38mm. The ball shall weigh 2.5g. The standard bounce required shall be not less than 23.5cm nor more than 25.5cm when dropped from a height of 30.5cm on a specially designed steel block. The standard bounce required shall not be less than 22cm nor more than 25cm when dropped from a height of 30.5cm on an approved table." The rebound height depends on elastic properties of the ball and the surface of impact, and is specified in dynamics by the coefficient of restitution.

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[1] For more information about table tennis see `http://www.usatt.org`.

1.2 System Setup

In the experimental setup, a microphone with an integrated pre-amplifier is attached to the sound card of a PC. A shareware software package, GoldWave[2], is used to record, play, and analyze the audio signal picked up from the microphone. The software displays the time history of the bounce acoustic signatures, from which bounce times are determined. (Sound measurement is addressed in mechanical measurements textbooks. See, for example, Section 18.5 of Beckwith, et al., 1993.)

2. THEORETICAL FOUNDATION

This experiment focuses on one type of collision which can be modeled classically as a direct, central impact of particles. The theory of direct, central impact of particles – which follows from Newton's laws of motion – can be found in numerous textbooks (e.g., Greenwood, 1988; Hibbeler, 1998) that present the fundamentals of dynamics. More advanced treatments of planar and three-dimensional impact for both particles and rigid bodies are developed in specialized textbooks (e.g., Brach, 1991). A method for the measurement of the coefficient of restitution for collisions between a bouncing ball and a horizontal surface is given by Bernstein (1977). The derivation of this paper draws heavily from this method.

A ball is dropped from known height h_o onto a tabletop, modeled as a massive, smooth, horizontal surface. The trajectory of the ball is depicted schematically in Figure 1 which shows the ball height as a function of time for the first few collisions. The ball is modeled as a particle, and vertical motion only is considered in the analysis. Due to the inelastic nature of the ball-table collision, the maximum height of the ball decreases successively with each impact. The trajectory of the ball can be described in terms of the ball height from the surface as a function of time. Neglecting aerodynamic resistance, for the simplest model, the flight time from the top of the trajectory to the surface is then half of the total flight time, T_n, between the n^{th} and $(n+1)^{\text{th}}$ bounce.

For the case of no air resistance, the vertical speed v_n (i.e., the vertical component of the velocity) of the ball associated with its n^{th} bounce is

$$v_n = g\left(\frac{T_n}{2}\right) \qquad (n = 1, 2, 3, \ldots) \tag{1}$$

where g is the acceleration due to gravity. Assuming

[2] The software can be downloaded from `http://www.goldwave.com`.

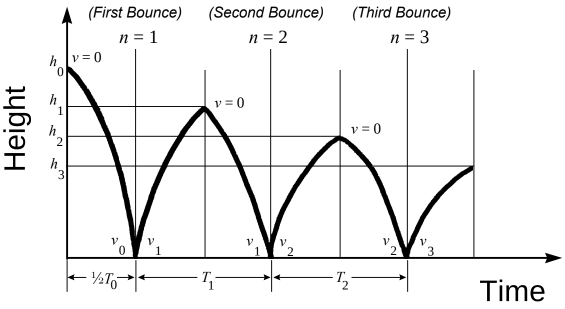

Figure 1. Graph of Ball Height versus Time for First Few Collisions

the coefficient of restitution, e, is constant

$$v_n = e v_{n-1} = e^n v_o \qquad (n = 1, 2, 3, \ldots) \tag{2}$$

Equating (1) and (2) yields

$$T_n = e^n\left(\frac{2v_o}{g}\right) \qquad (n = 1, 2, 3, \ldots) \tag{3}$$

from which

$$\log T_n = n\,\log e + \log\left(\frac{2v_o}{g}\right) \qquad (n = 1, 2, 3, \ldots) \tag{4}$$

Plotting equation (4) in a graph of $\log(T_n)$ vs. n yields a straight line, whose slope is log e and whose ordinate intercept can be used to determine v_o from which the initial height h_o can be found. In general, the height h_n reached after the n^{th} bounce can be written as

$$h_n = \frac{v_n^2}{2g} \qquad (n = 0, 1, 2, \ldots) \tag{5}$$

from conservation of energy.

Although the number of bounces is infinite in theory (assuming no air resistance), the total time required for the ball to come to rest and the total distance traveled are both finite. (See Example 4-5, Greenwood, 1988.) The total time required for the ball to come to rest is

$$\begin{aligned} T_{total} &= \frac{1}{2}T_o + \sum_{n=1}^{\infty} T_n = \frac{v_o}{g} + \left(\frac{2v_o}{g}\right)\sum_{n=1}^{\infty} e^n \\ &= \frac{v_o}{g}\left[1 + 2\sum_{n=1}^{\infty} e^n\right] \\ &= \left(\sqrt{\frac{2h_o}{g}}\right)\left[1 + 2\sum_{n=1}^{\infty} e^n\right] \\ &= \left(\sqrt{\frac{2h_o}{g}}\right)\left(\frac{1+e}{1-e}\right) \end{aligned} \tag{6}$$

The total distance traveled along the path is

$$s_{total} = h_o + 2\sum_{n=1}^{\infty} h_n$$

$$= h_o\left[1 + 2\sum_{n=1}^{\infty} e^{2n}\right] = h_o\left(\frac{1+e^2}{1-e^2}\right) \qquad (7)$$

3. PROTOCOL

The procedure is to drop the ball from rest from a measured height above the table such that it lands and bounces near the microphone. The software is configured for automatic recording once a bounce sound is detected. Bounce sounds are indicated by spike amplitudes in the audio signal, as shown in Figure 2, and the times associated with the bounces are found. From a linear regression curve fit of the $\log(T_n)$ vs. n graph, as plotted in Figure 3, the coefficient of restitution e and height of initial ball drop h_0 can be determined. (The governing relations are equation (4) and the n=0 case of equation(5).)

Students are asked to address the following sample questions in their analysis:

1. Determine the coefficient of restitution of a ball with an "approved table" based on the rules of table tennis. Compare this with the experimentally determined coefficient.
2. Determine the initial height h_o from which the ball was dropped as well as the height h_n after the n^{th} bounce. Compare the calculated initial height with the measured height.
3. Determine the total time required for the ball to come to rest and the total distance traveled along the path. Comment.
4. Investigate the assumption that air resistance is negligible. Determine the coefficient of restitution neglecting the first ten or fifteen bounces. Is this coefficient of restitution more accurate than the one accounting for the full bounce history? Develop a model that includes aerodynamic resistance and simulate its effect.
5. Investigate the assumption that the coefficient of restitution is a constant. In particular, consider the possibility that the coefficient of restitution is a function of approach speed. Calculate e separately for each collision directly from the definition, $e_n=v_n/v_{n-1}$, plotting the results as a function of approach speed.

4. RESULTS & DISCUSSION

A ping-pong was ball dropped from an initial height of 0.35 m. The coefficient of restitution e and the height of the initial ball drop h_0 were calculated from the linear regression curve fit of Figure 3 (plotted as the natural logarithm of flight time vs. bounce number) giving e = 0.952 and h_0 = 0.286 m. The difference in predicted vs. actual initial height is attributed to neglecting aerodynamic drag in the analysis.

The experiment engages students in creative thinking about the dynamics of a simple impact problem, and brings to life equations of physics. With basic instructions provided in a laboratory manual, the experiment has proven to be "open-ended". Students are encouraged to explore different impact situations, and test their suspicions regarding the effect of height and noise of collision with the change in coefficient of restitution.

5. CLOSING

This experiment has been incorporated into a junior-level mechanical engineering course, MEEN 120 "Mechanical Measurements and Instrumentation" at Marquette University. The experiment has minimal requirements for hardware (PC with sound card, microphone, ball), software (GoldWave shareware), and time (taking approximately 5-10 minutes to

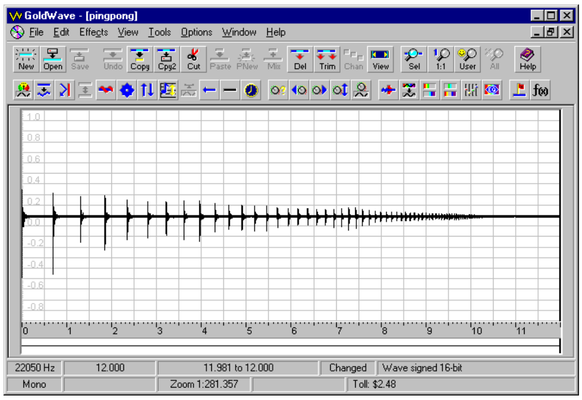

Figure 2. Sample Audio Signal of Ball Bounces

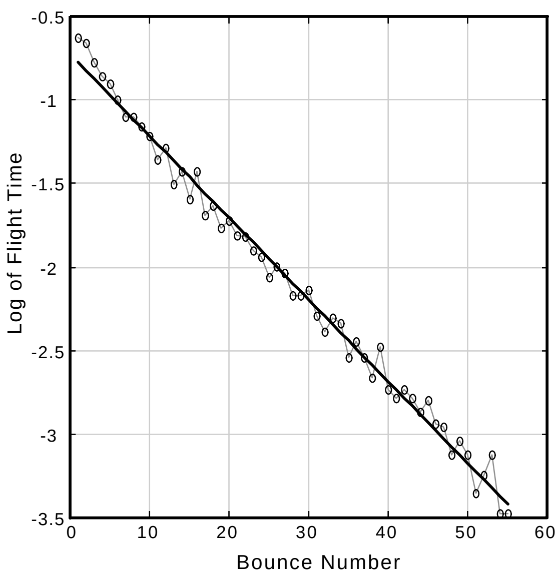

Figure 3. Log of Flight Time vs Bounce Number

conduct). Since it does not require any special laboratory facilities, the experiment can be conducted in a classroom using a notebook computer with a sound card and microphone. The experiment has been well received, has fostered significant discussion with students, and is suggested for use in courses in "measurements" and "dynamics".

ACKNOWLEDGEMENT

The contributions of Ms. Carol (Kai) Liu, who served as a teaching assistant in "MEEN 120: Mechanical Measurements and Instrumentation" at Marquette University, are gratefully appreciated. The author also wishes to thank his department and, in particular, the Associate Dean of the College of Engineering, Dr. G.E.O. Widera, for his technical input and generous support of our laboratory activities.

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