

## Book Review

*Probabilistic Models for Dynamical Systems, Second Edition*, by Haym Benaroya, Seon Mi Han and Mark Nagurka CRC Press/Taylor & Francis Group, 2013; ISBN-13: 978-1-4398-4989-7 (hardback), USD 119.95

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A probabilistic model is a model that uses probability theory to describe uncertainty in data. Probabilistic methodology is widely used to model reality; applications range from quantum field theory, statistical physics, and turbulence theory to engineering sciences.

The book consists of fourteen chapters. Many problems and examples are included, some of which are solved in detail. The authors also include historical notes throughout the text giving brief summaries of the history of probability theory and the main researchers in analysis, set theory, probability theory, and dynamic systems theory (Leibniz, Newton, Poisson, Gauss, Khinchine, Kolmogorov, Wiener, Markov, von Neumann, Poincaré, Kálmán, etc.).

The book is an excellent introduction to probability for engineers, applied geophysicists, and practitioners. It begins with a discussion of some applications including random vibrations of structures, the effect of wave-induced forces on offshore platforms, and several engineering structures subjected to wind interactions.

The second chapter develops the elements of the algebra of sets needed to present axiomatic probability theory. Presentation of the basic facts is complemented by practical interpretation of probability as a frequency of occurrence. The notions of statistical independence, conditional and total probability, and Bayes' theorem are also explained briefly.

Chapter 3 covers random variables and the machinery for dealing with them—distribution functions, probability density functions, expected values, moment-generating functions, characteristic functions, etc. Five continuous probability density functions of particular importance for applications are introduced and discussed. The uniform, exponential, normal (or Gaussian), lognormal, and Raleigh density functions are included. Among these, the normal distribution is the most important because many phenomena involve random variables that have the characteristics of the Gaussian distribution. Also, two important discrete probability density functions connected with the binomial distribution and the Poisson distribution are briefly discussed. Finally, the statistical characteristics of two random variables are described briefly.

In Chapter 4, constructive methods are developed to enable the densities of derived functions to be obtained on the basis of densities that are given. First, the analytical approaches are presented, including the parabolic transformation, the harmonic transformation, product mapping, and quotient mapping. Unfortunately, in many instances, only approximate and numerical methods are available. Such techniques are presented and used to obtain the approximate densities and approximate statistical data.

Chapter 5 deals with random (or stochastic) processes. The basic notions are introduced, namely, ensemble averaging, stationarity, ergodicity, and Fourier transformation, among others. This enables consideration of the important class of stationary processes and derivation of an expression for the energy power spectrum. This is then used to obtain

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the power spectrum of a harmonic process by use of Wiener–Khinchine relationships. The Wiener–Khinchine theorem is also briefly discussed. Spectacular examples include ocean engineering, earthquake physics, offshore structural dynamics, machine dynamics, and crack dislocation in materials. The chapter concludes by considering two very important types of process, narrow and broad-band processes, with information about white noise and coloured noise processes, with physical interpretations of correlations and spectra.

Chapter 6 is devoted to the vibration of a single degree-of-freedom system, i.e. a system for which the motion is determined by the time variation of a single coordinate. The basic equation describing the motion of a single body with mass, linear stiffness, and viscous damping under the effect of external forces, is derived first. Free vibrations, forced vibrations, and damped vibrations are then analysed. Next, base excitation problems are formulated, leading to the convolution (or Duhamel) equation, the frequency response function, and the impulse response function. This enables derivation of general formulae for the response statistics of a system oscillating as a result of a random force. Furthermore, the relationship between output spectral density, input spectral density, and system frequency response function is derived and discussed. Some generalizations regarding systems subjected to two random forces are also briefly discussed.

Chapter 7 continues the study of vibration. The analysis is extended to multi degree-of-freedom systems subjected to random forces. The first section provides a general overview of deterministic vibration theory for an  $N$  degree-of-freedom system. Next, two investigation techniques are described: the impulse response function method and the modal analysis approach. Finally, three special issues are considered: periodic structures, inverse vibration, and random eigenvalues.

Chapter 8 concentrates on continuous system vibration. If a system is modelled as a continuous system, the governing dynamic equations are partial differential equations, which are more difficult to analyse. Three generic continuous systems are investigated: taut string, axially vibrating beam, and transversely vibrating beam. Direct and modal

solution techniques are used and then generalized to include random forcing. A specific type of boundary condition is chosen to demonstrate how complicated physical problems can be approximated by use of such model structures as beams.

The notion of the reliability of a (mechanical) system or component is defined as the probability of operating under prescribed conditions for a specified period of time. Chapter 9 presents basic information enabling determination of the reliability and maintainability characteristics of a mechanical system subjected to random disturbances. Two types of failure are considered—excursion failure and fatigue failure. The four failure laws, the exponential, gamma, normal, and Weibull distributions, are analysed and discussed. Fatigue life is assessed by use of the stress-based fatigue failure rule (Miner's rule).

Mathematical modelling of real-world processes leads, in general, to nonlinear deterministic and stochastic dynamic systems. Chapter 10 investigates periodic solutions of nonlinear single degree-of-freedom oscillators connected by nonlinear vibrations. The transcendental, the Duffing, and the van der Pol equations are derived, and briefly analysed and discussed. The phase portrait method and perturbation techniques are used and explained. For the Duffing and van der Pol oscillators, the Markov processes approach is also discussed. By applying Markov process-based concepts and methods the Chapman–Kolmogorov equation and the related Fokker–Planck equation are methodically derived for appropriate situations.

Chapter 11 concerns non-stationary stochastic modelling. Three motivating geophysical themes are considered: jet noise spectra, seismological data, and ground motion. To illustrate the problems, several evolutionary-type approaches are reviewed: envelope function model, matrix formalism, and the technique of equivalent linearization. The Fourier–Stieltjes representation of the random process is discussed, and used to calculate a variety of oscillator response functions.

Chapter 12 gives a brief overview of Monte Carlo methods. Starting with random number generation, several procedures for generation of nonuniform random variables in accordance with prescribed probability density functions, for example the inverse

transform method, the composition method, and Von Neumann's rejection-acceptance method, are elucidated. The problem of error estimation in Monte Carlo integration is also presented.

Chapter 13 focuses on fluid-induced vibration. Special attention is given to description of fluid forces arising as a result of random ocean waves and currents. The elements of spectral wave theory are briefly discussed and several types of fluid force are covered. Four practical examples including towing cable, articulated tower, wave height statistics, and wave elevations are presented.

Chapter 14 concludes the book by describing probabilistic models in control and mechatronic systems. The mathematical apparatus for study of linear, continuous-time models of probabilistic systems is developed. A control theory of deterministic dynamic systems is presented first. The concepts of stochastic systems are then introduced. The general filtering

problem is formulated and the conditions under which the general filter simplifies to a Kálmán filter, one of the most important achievements in control theory, are explained.

The book is clearly written, very well-organized, and supported with excellent figures. Problems at the end of each chapter enable the reader to monitor the process of learning. I strongly encourage readers to do the problems because these subjects help give students deeper understanding of statistical aspects of the phenomena presented. The book can also be recommended as an appropriate textbook for undergraduate and graduate courses on the subject.

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