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A SUBOPTIMAL CONTROL APPROACH FOR THE STUDY OF BIPEDAL LOCOMOTION

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ABSTRACT

This research focuses on an approach to study optimal motions of limb segments of a planar mechanical walking model. The major advantage of this approach is that simulated optimal motions of the limbs are a function of a performance index (such as mechanical energy) and physically-based system constraints. A Fourier-based approximation technique is used to convert the optimal control problem into a nonlinear programming problem, which is then solved for the suboptimal histories of joint angles, velocities, accelerations, and torques. By investigating different performance indices and comparing the resulting motion histories with human walking data, the method hopefully can be used to study strategies that humans use in selecting dynamic patterns of limb motions.

INTRODUCTION

Three different approaches that have been used to study bipedal locomotion are the direct dynamic, the inverse dynamic, and the optimal control methods. Each of these approaches assumes the existence of a mathematical model of bipedal locomotion.

The direct dynamic approach views generalized forces (joint torques) as system inputs. By solving the equations of motion of the bipedal locomotion model, the time history of generalized coordinates (joint displacements) and their derivatives (joint velocities and accelerations) can be obtained. Since the generalized coordinates predicted via the direct dynamic approach often differ from those of the real system, some physically-based constraints satisfied during normal bipedal locomotion are often violated.

The inverse dynamic approach uses the time history of the generalized coordinates and their derivatives as inputs. From the equations of motion, the corresponding generalized forces can be obtained. This approach is computationally efficient and robust to modeling errors and has been widely used in bipedal locomotion research.

The major problem of the inverse dynamic approach is its sensitivity to measurement noise associated with the generalized coordinates. Typically, joint displacements are measured, and joint velocities and accelerations are obtained by differentiation. Differentiation of measurement noise introduces errors which limit the accuracy of this approach.

The optimal control approach assumes a "principal of optimality," such as minimum energy expenditure during human locomotion. The optimal control approach has the advantage that it reduces dependency on experimental data, but is fraught with several difficulties. The first problem is associated with the determination of the performance index. For instance, there is no clear way to express biological characteristics of the human body, such as energy consumption, analytically. The second problem is related to computer implementation Standard optimal control algorithms are typically very sensitive to numerical errors and/or require very large amount of computer memory. As a result, previous optimal control approaches applied to bipedal locomotion have been limited to the use of single leg models, which do not account for interaction between legs and usually consist of two links [1,2]. The implementation of the optimal control approach for high-order, nonlinear bipedal models remains a research challenge.

Scope. This paper proposes an optimal control approach for determining segment trajectories of a five-link, planar, bipedal locomotion model. The approach is based on a Fourier-based approximation method that guarantees an optimal solution. Thus, this research extends earlier studies limited to single leg models, and is aimed toward implementing the optimal control approach for higher order, nonlinear models.

METHODOLOGY

The human musculoskeletal system is represented by a five-link rigid-body model with ideal torque actuators at the joints. The head, arm, and truck (HAT) are modeled by a single link. Each leg has two links, representing the thigh and shank. A massless foot is attached rigidly to the shank.

To simulate bipedal locomotion, the mechanical model must satisfy physically-based system constrained, five of which are identified below:

1. Desired Forward Speed. The horizontal speed of a point on the upper body link is constrained to be a known function of time. Here, it is assumed that the uppermost point of the HAT link is a known constant forward velocity.

2. Desired Upper Body Attitude. The attitude angle of the HAT link is constrained to be a known function of time. Here, it is assumed that the link is perfectly vertical during each step.

3. Relative Joint Displacements. The joint displacement of the thigh is constrained to be larger than the joint displacement of the shank of the same leg.

4. Double Stance Leg Displacements. During double stance, the feet have zero vertical displacement and are constrained to a fixed horizontal displacement (stride).

5. Swing Leg Clearance. The foot of the swing leg must clear (i.e., not go under) the ground.

The objective is to solve for the optimal trajectories of the bipedal locomotion model with the constraints identified above. Initial attempts to solve this type of problem by a standard optimal control approach (based on the calculus of variation technique) met with major numerical difficulties. To overcome these problems, a suboptimal control approach was developed that converts the optimal control problem into a nonlinear programming problem by using a Fourier-based approximation technique.

The approach assumes the existence of a performance index (an integral in terms of generalized coordinates and forces) and system constraints. The time response of each generalized coordinate is represented by the sum of a fifth-order auxiliary polynomial and a finite number of terms of Fourier-type expansion functions. The generalized coordinate rates which appear in the equations of motion are obtained by direct analytical differentiation. The generalized forces are then calculated directly from the equations of motion. The performance index of the optimal control problem can be evaluated by a standard numerical integration method.

The problem then becomes an algebraic nonlinear programming problem with the coefficients of the Fourier-type functions and the free boundary conditions as unknowns. Typically, these unknowns are found by standard iterative techniques of nonlinear programming. In every iteration, the coefficients of the auxiliary polynomials are determined such that they satisfy the boundary condition requirements and the system constraints.

The effectiveness of the suboptimal control approach has been tested by computer simulation studies. It has been found that satisfactory results can usually be achieved by using two or three terms of the Fourier-type expansion functions.

CONCLUSIONS

This paper proposes an optimal control approach for studying bipedal locomotion. A Fourier-based approximation scheme for obtaining the suboptimal trajectories of a bipedal model has been formulated. This scheme offers a new means to handle high order, nonlinear bipedal models, which previous optimal control studies have not investigated.

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