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SUBOPTIMAL TRAJECTORY PLANNING OF A FIVE-LINK HUMAN LOCOMOTION MODEL

V. Yen and **M.L. Nagurka**
Graduate Assistant *Assistant Professor*

Department of Mechanical Engineering
and The Robotics Institute
Carnegie-Mellon University
Pittsburgh, PA 15213

ABSTRACT

A methodology designed to help uncover strategies that play a role in the coordinated management of limb segment motions during human locomotion is proposed. The methodology involves solving for the optimal segment trajectories of a five-link bipedal locomotion model, where the segment motions are a function of a performance index (such as control effort) and physically-based system constraints (such as ground clearance during swing). To avoid numerical difficulties, the optimal control problem is converted into a nonlinear programming problem by representing the joint displacements by the sum of a polynomial and a set of appropriate eigenfunctions. Upon application of a direct search optimization algorithm, the suboptimal histories of joint angular displacements, rates, and torques can be obtained. An important advantage of the approach is related to the streamlined nature of solving the optimal control problem enabling the simulation of high order bipedal locomotion models. An additional advantage of the approach is that it minimizes the requirements for experimental data, although it is flexible in utilizing available data. Results of simulation studies demonstrate the utility of the approach for studying the dynamics of the interaction of the legs for the first step.

INTRODUCTION

The effective utilization of different locomotion patterns that humans exhibit in moving about the environment is a marvelous feat of nature. The resulting dynamic motions of the limb segments during locomotion have been the subject of many previous scientific investigations including [1-6]. These studies generally adopt a methodology such as the direct dynamic, inverse dynamic, or optimal control approach. Each of these approaches starts with a mathematical model of the human body, which is usually represented by a system of differential equations.

The direct dynamic approach views joint torques (or more generally generalized forces), which typically are obtained experimentally, as system inputs. By numerically integrating the model equations of motion, the time history of the joint displacements (or more generally generalized coordinates) can be obtained. These displacements are treated as system outputs. The direct dynamic approach often fails due to modeling errors, *i.e.*, discrepancies due to simplifications of the mathematical model relative to the actual system. As a result, when measured system inputs are used to "drive" the mathematical model, constraints associated with normal bipedal locomotion (*e.g.*, swing leg clearance) are often violated. The validity of the simulated motions of the model is oftentimes questionable.

The inverse dynamic approach, on the other hand, views the time history of the joint displacements and their derivatives as inputs. From the equations of motion, the corresponding joint torques can be obtained readily. This approach is very efficient computationally and is much less sensitive to modeling errors than the direct dynamic approach. It is perhaps for this reason that it has been widely used in bipedal locomotion research.

The major problem of the inverse dynamic approach is its sensitivity to measurement noise which is always present in experimental data. When applying the inverse dynamic approach, the actual joint displacements are measured, and typically the joint velocities and accelerations are obtained by differentiation techniques (numerically, graphically or electronically). Although many smoothing or filtering methods (*e.g.*, [7,8]) have been developed to reduce the error associated with the differentiation of the measurement noise, the accuracy is still limited [9].

The optimal control approach usually endorses a "principle of optimality." Typically, minimum energy expenditure during locomotion is assumed, an assumption based on experimental results showing that self-determined walking patterns minimize oxygen consumption [10-12]. Once a performance index and system constraints are identified, the optimal trajectory of a model of known parameters can be determined. The optimal control approach does not require explicit joint kinematic data (needed for the inverse dynamic approach) nor joint kinetic data (needed for the direct dynamic approach). There are, however, two principal difficulties in implementing the optimal control approach. One difficulty is associated with selecting (and expressing analytically) the performance index. For example, even if energy is the selected criterion, it is not clear how to express it analytically for the human body. A second difficulty is a numerical problem. Generally, standard optimal control algorithms are very sensitive to computational errors and/or require a very large amount of computer memory. As a result, previous locomotion studies based on optimal control methods have been limited to simple models, such as single leg models [2,5]. The application of optimal control approaches to high order nonlinear systems, such as a relatively more complete bipedal locomotion model, motivated the current research.

This paper describes a methodology for simulating optimal segment trajectories of a planar, five-link, locomotion model. The approach avoids many of the implementation difficulties encountered in applying optimal control theory to high order, nonlinear models. In addition to formulating the problem, computer studies focusing on the initial step of simulated locomotion are reported.

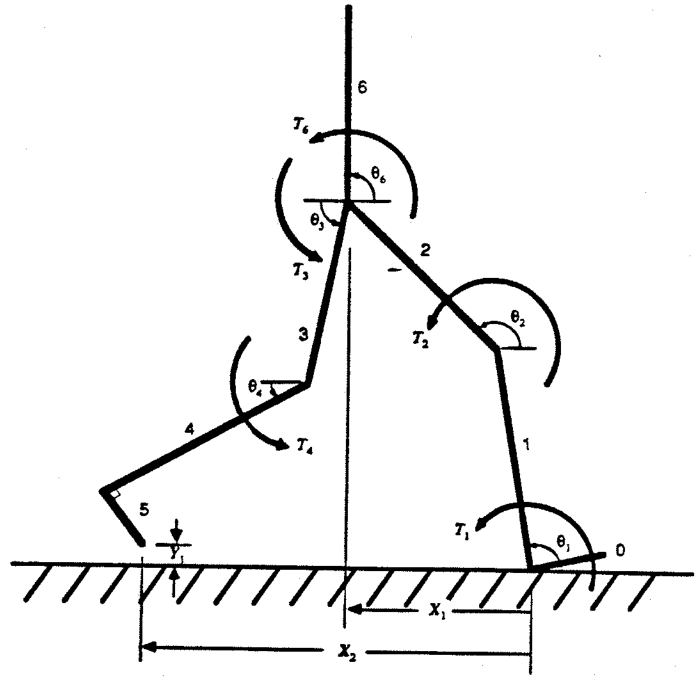

Figure 1: Bipedal Locomotion Model.

BACKGROUND

An early attempt to formulate a principle of optimality for a muscle-driven system was reported by Nubar and Contini [13]. They assumed that the self-determined walking pattern minimizes the total "muscle effort," which they defined as a product of a constant, the square of the joint torques, and the period of the walking cycle. Nubar and Contini did not simulate optimal trajectories of their locomotion model; optimal control theory was still in its infancy.

With the appropriate mathematical tools, Chow and Jacobson [2] formulated the bipedal locomotion problem as an optimal control problem and obtained optimal trajectories for their model. They first employed a five degree-of-freedom rigid body model representing the trunk, the thighs and the shanks. The feet were considered massless. Certain kinematic constraints, including clearance of the foot during swing, were also included. In their formulation, the sum of the joint torques squared was proposed as the performance index.

In order to simplify their problem, Chow and Jacobson ignored interaction between the legs, and reduced their five degree-of-freedom model to a two degree-of-freedom model consisting of two links. They also approximated the vertical hip trajectory and the ankle trajectory by harmonic functions of polynomials which they derived from experimental data. Although these functions appear to be reasonable approximations, it is not clear whether their derivatives (which, along with the functions, are explicit variables in the performance index) are acceptable. Finally, in their approach the initial and terminal conditions of the joint variables are experimentally based. As a result, their method can only be applied if such data are available.

Instead of trying to predict the optimal trajectories for normal locomotion, Hatze [5] studied a time-optimal problem for a two-link model. He conducted computer simulation studies and an experiment in which a human subject wearing a special heavy shoe was asked to walk as fast as possible while hitting with his foot appropriately positioned targets.

Hatze successfully showed that the simulated trajectories and time period matched quite closely those of the actual locomotion. However, the applicability of this work is limited since normal human locomotion is typically not a time optimal problem. Consequently, this method is not useful for predicting the general dynamic characteristics of human locomotion. Also, as with the method described by Chow and Jacobson, the approach depends heavily on experimental data.

The current research develops a methodology to generate the optimal trajectories of a planar, five link, rigid body model that accounts for the interaction between the legs and minimizes the requirements for experimental data.

MODEL SPECIFICATION

In this study, the human musculoskeletal system is modeled by a system of rigid articulated links with associated actuators at the joints. The head, arm and trunk (HAT) are represented by one rigid link. Each leg has two links which represent the thigh and shank. A massless foot is attached to the shank with a rigid ankle. The linkage model is sketched in Figure 1, which also shows the joint torque actuators, representing the effect of muscle forces, which are used to drive the model. The dimensions and mass inertial properties of all links are assumed known, and their values for the simulation studies are given in the Appendix.

The influence of the arm dynamics has been neglected since the HAT is only represented by a single link. However, it has been reported [10,11] that representing the HAT by one segment is a reasonable assumption when the arms do not swing excessively.

Kinematic Constraints

This section identifies some kinematic constraints associated with normal locomotion. These constraints capture some of the essential characteristics of the locomotion and/or are introduced to simplify the problem. The displacements X_1, X_2, and Y_1, which appear in the following paragraphs, are identified in Figure 1.

1. **Desired Forward Speed:** $\dot{X}_1$ = constant walking speed for the entire walking cycle.

   The model is assumed to move at a constant forward speed corresponding to the speed of normal locomotion.

2. **Upper Body Attitude:** $\theta_6 = 90°$ for the entire walking cycle.

   As reported by [14], the maximal excursion of the head and shoulder points with respect to the pelvis point along the antero-posterior axis is relatively small (approximately 20 mm.) Thus, it is reasonable to assume that the HAT is fixed rigidly in a vertical position.

3. **Relative Joint Displacements Between the Two Leg Joints:** $\theta_2 \geq \theta_1$ and $\theta_3 \geq \theta_4$ for the entire walking cycle.

   These inequalities guarantee that the joint displacement of the thigh is not less than the joint displacement of the shank, a feature of normal locomotion that can be verified by direct observation.

4. **Swing Foot Clearance:** $Y_1 \geq 0$ for the single stance phase.

   Y_1 represents the vertical clearance of the swing leg. This constraint ensures that the foot of the swing leg clears the ground.

5. **Coordination of Legs in Double Stance Phase:** $\dot{X}_2(t) = \ddot{X}_2(t) = 0$ for the double stance phase, and $Y_1(t) = \dot{Y}_1(t) = \ddot{Y}_1(t) = 0$ for the double stance phase (including the time at which the single and double stance phases coincide.

   Here, X_2 is the horizontal distance between the pivoting points (or centers of pressure) of both feet. These constraints are due to the fact that during double stance phase the human body pivots on both feet.

These system constraints as well as the model must be specified before the method for predicting optimal trajectories can be applied. The method, which is developed below, minimizes any well defined performance index (which is a function of the model variables and parameters) without violating any of the system constraints that have been imposed.

OPTIMAL TRAJECTORY GENERATION ALGORITHM

A general form of the performance index is

$$J = \int_0^{t_f} g(\underline{\theta}(t), \dot{\underline{\theta}}(t), \underline{T}(t), t)\,dt \qquad (1)$$

where g represents a general function and $[0, t_f]$ is the time interval, for instance, the time of the walking cycle. If the performance index is the energy consumption, then g represents the total mechanical power, *i.e.*, the sum of the products of the joint torques and angular speeds of the bipedal locomotion model. The goal of the optimal control method is to find an admissible control, $\underline{T}^*$, that causes the model to follow an admissible trajectory, $\underline{\theta}^*$, $\dot{\underline{\theta}}^*$, and $\ddot{\underline{\theta}}^*$, such that the performance index is minimized without violating the constraints.

The basic idea of the suboptimal approach is to approximate each joint angular displacement by the sum of a fifth order polynomial and finite terms of a Fourier-type series. For example, for joint i,

$$\theta_i(t) = P_i(t) + F_{ki}(t), \qquad (2)$$

where the auxiliary polynomial, $P_i(t)$, is

$$P_i(t) = p_{i0} + p_{i1}t + p_{i2}t^2 + p_{i3}t^3 + p_{i4}t^4 + p_{i5}t^5 \qquad (3)$$

whose coefficients are determined to satisfy the boundary conditions requirements. The K term Fourier-type series is defined as

$$F_{ki}(t) = \sum_{k=1}^{K} a_{ik} \cos\frac{2k\pi t}{t_f} + \sum_{k=1}^{K} b_{ik} \sin\frac{2k\pi t}{t_f} \qquad (4)$$

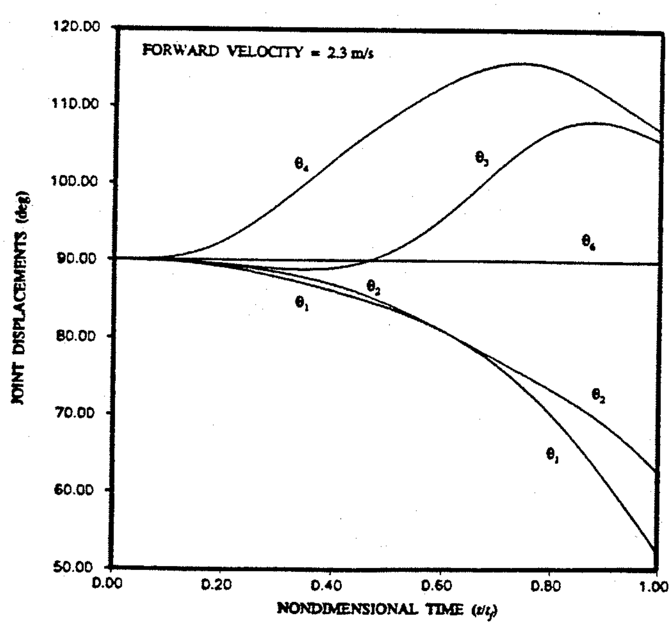

Figure 2: Optimal Joint Displacement History for First Step.

The angular velocity and acceleration of the i-th joint are obtained by direct differentiation of the above equations. This approach is adopted for each joint, and then the control variables, *i.e.*, the joint torques, are calculated readily from the equations of motion. Finally, the performance index is computed using a straightforward numerical integration method such as Simpson's composite integral technique.

In essence, this technique converts the optimal control problem into a nonlinear programming (NLP) problem, which can be solved using well-developed NLP algorithms. Here, the NLP (subject to constraints) is solved using a Simplex algorithm for the suboptimal histories of joint displacements, velocities, accelerations, and torques. The free variables in the optimization are the free boundary conditions, the coefficients of the Fourier-type series, and the walking cycle time, t_f. In the simulation studies below, two term Fourier-type series are used (*i.e.*, $K = 2$), and thus there are four free coefficients per joint. Since only finite terms are included, the method is called a **suboptimal** trajectory planning method. A more complete description of the suboptimal method can be found in [15].

SIMULATION

It is assumed that the model starts at rest from a vertical position with initial joint displacements θ_1, θ_2, θ_3 and θ_4 equal to 90°. Furthermore, it is assumed that at the end of the first step (*i.e.*, at time t_f), the model reaches a desired forward speed with which the model will proceed in subsequent steps. (Note that kinematic contraint 1 does not apply for the first step.) The period, t_f, is assumed to be free.

The initial conditions of the second and subsequent steps coincide with the terminal conditions of the first step (*i.e.*, the model is forced to return to its first step terminal configuration.) The performance index is the total energy consumption of all steps divided by the distance travelled, where the energy consumption is represented by the sum of the square of joint torques as proposed in [2]. In selecting this performance index, the goal is to generate the trajectories that achieve optimal locomotion efficiency represented by minimum energy consumption per unit distance.

Simulations for desired forward speeds ranging from 1.0 m/sec to 2.5 m/sec have been conducted. The results for a speed of 2.3 m/sec are reported here. Figure 2 shows the optimal joint angular displacements as a function of nondimensional time, defined as t/t_f, the ratio of the real time to the period, where the (sub)optimal period was determined to be $t_f = 0.61$ sec. The displacement of the HAT, θ_6, is fixed at 90°, as specified by constraint 2. The displacement of each shank is less than or equal to the displacement of the corresponding thigh, in agreement with constraint 3. The displacements θ_1 and θ_2 are approximately equal for more than half of the step.

Figure 3 shows a multiple exposure schematic of the model (without the HAT link) during the first step, and is actually an alternative representation of Figure 2. It can be observed that the model initially moves slowly, and accelerates to its position at the end of the first step to meet the target terminal speed. Figure 3 also demonstrates the satisfaction throughout the step of the swing leg clearance, *i.e.*, constraint 4.

The optimal joint torque histories are presented in Figure 4. The figure shows that for optimality a complicated coordination

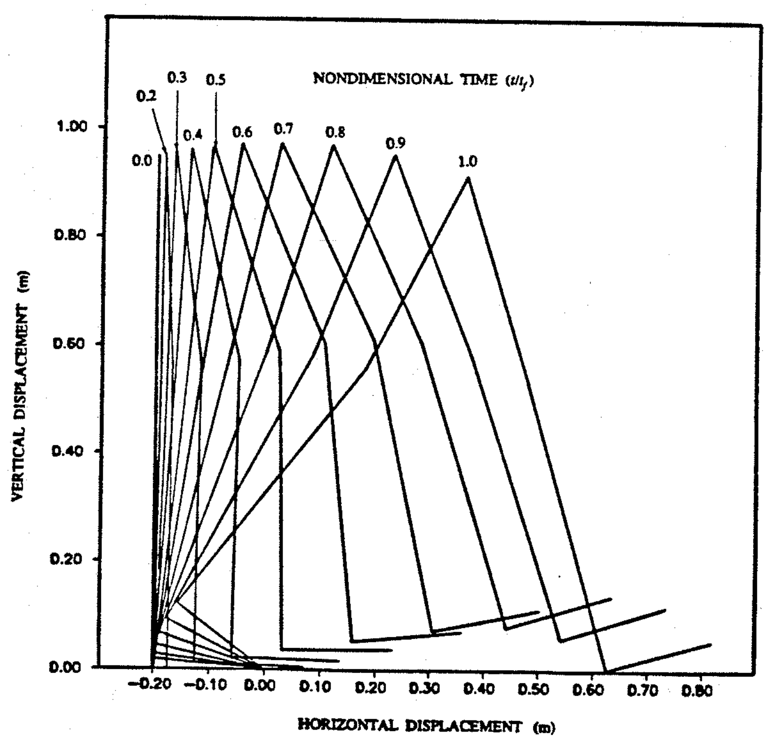

Figure 3: Multiple Exposure Schematic of Model for First Step.

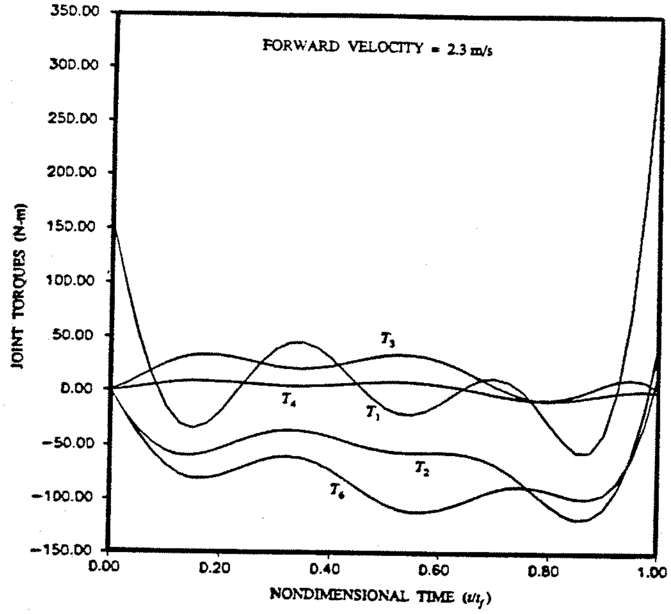

Figure 4: Optimal Joint Torque History for First Step.

among the joint torques is necessary. The torques of the stance leg, T_1, T_2, and T_6, are larger in magnitude than the torques of the swing leg, T_3 and T_4. As such, the model is driven principally by the stance leg. The maximum torque is developed by T_1, the torque at the ankle, at the end of the step. The relatively small joint torques of the swing leg suggest that the swing leg behaves like a ballistic double pendulum. The torques of smallest magnitude are developed at T_4, indicating that the knee joint of the swing leg plays a minimal role during the first step of locomotion.

SUMMARY

This paper develops and formulates an optimal control approach suitable for high order bipedal locomotion models. Here, the proposed approach provides a tractable scheme for determining optimal trajectories of a five link bipedal model.

The advantages of this approach can be summarized as follows: (i) By leaving both the terminal time and the terminal states free, the requirements for experimental data are minimized. (ii) By considering the specified model, the dynamics of the interaction between legs can be investigated. (iii) With minimum analytical and programming effort, the trajectories corresponding to different performance indices can be explored.

This work can be extended by applying the proposed approach to determine the trajectories of a complete walking cycle, including the double stance phase. Furthermore, important dynamic characteristics such as the interchange between kinetic and potential energies can be studied.

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APPENDIX

NOMENCLATURE:

D_i: Distance between the intersection of link i-1 and i and mass center of link i.

L_i: Length of link i.

M_i: Mass of link i.

I_i: Moment of inertia of link i.

PARAMETER VALUES:

$D_1 = 0.2900$ (m)
$D_2 = 0.2270$ (m)
$D_3 = 0.1730$ (m)
$D_4 = 0.2600$ (m)
$D_6 = 0.3250$ (m)

$L_0 = L_5 = 0.2000$ (m)
$L_1 = L_4 = 0.5500$ (m)
$L_2 = L_3 = 0.4000$ (m)

$M_0 = M_5 = 0.0$ (kg)
$M_1 = M_4 = 4.8760$ (kg)
$M_2 = M_3 = 7.9940$ (kg)
$M_6 = 54.200$ (kg)

$I_0 = I_5 = 0.0$ (kg-m^4)
$I_1 = I_4 = 0.1350$ (kg-m^4)
$I_2 = I_3 = 0.1330$ (kg-m^4)
$I_6 = 3.5910$ (kg-m^4)