Inelastic Behavior of Multistory Partially Restrained Steel Frames. Part I

By Christopher M. Foley,[1] Associate Member, ASCE, and Sriramulu Vinnakota,[2] Fellow, ASCE

Abstract: This paper outlines the development of a finite element for use in the inelastic second-order analysis of partially (PR) and fully (FR) restrained planar steel frames. The finite element considers spread of plastification within the cross section and along the member length; centroidal shift of the elastic core; a variety of residual stress distributions; and *P*-Δ effects. Nonlinear spring elements are used in modeling beam-end connections. An arc-length technique (with iteration) is used in the analysis of individual beam-columns, while constant-work simple Euler stepping (no iteration) is used in the nonlinear analysis of frameworks. Full nonlinear load-deformation response of planar beam-columns and frames is computed. Accuracy is demonstrated through comparison with previously published analytical results. It is shown that concentrated plasticity models may lead to an overestimation of connection rotation in PR frameworks. A companion paper discusses the inelastic ultimate load analysis of large steel PR frameworks and evaluation of connection classification systems.

INTRODUCTION

The inelastic behavior of steel frames has been studied extensively since the introduction of load and resistance factor design (*Load* 1993). Recently, researchers have begun incorporating the presence of partially restrained connections in the inelastic analysis of steel frames. The most common approach used in the ultimate load analysis of steel frameworks is based on the assumption that concentrated plastic hinges form at the ends of a member. An additional method, termed spread of plasticity (plastic zone), models the beams and columns with many finite elements. Furthermore, the cross sections of each element are broken down into ''fibers'' in which the stresses and strains are monitored. Thus, the spread of plasticity model allows plastification to be tracked along the member length and within the cross section. Also, residual stress patterns are easily and directly included in the analytical model using the fiber-element concept. The reader is referred to a published report (ASCE 1990) for a thorough description of research related to structural steel column stability prior to 1984.

Researchers have recognized that simple plastic-hinge-based analytical models have limited accuracy depending on framework configuration and loading characteristics. Vinnakota (1972) indicated that the lower stories in high-rise structures may have columns in which yielding occurs over the entire length of the member. Furthermore, King et al. (1990) have shown that large axial forces within the members (columns) of a framework can cause yielding along the length, which is not accounted for in classical concentrated plasticity approaches. Plastic-hinge- and modified plastic-hinge-based analytical methods have been shown to adequately capture the ultimate load of fully restrained (FR) frameworks. However, classical plastic-hinge methods do not directly assign residual stresses within the cross section, do not keep track of strains throughout the cross section for determination of local buckling tendencies, and are suspect in modeling the spread of plastification along the length of beams and columns in large-scale partially restrained (PR) frames. It is generally recognized, that the plastic zone (spread of plasticity) method is the most accurate means for analyzing many aspects of frame behavior, and it is used to establish calibration frames (Vogel 1985).

Although the plastic-zone method of analysis is more computationally intensive than one-element-per-member plastic-hinge analysis, many researchers have used the technique to analyze small FR frames and individual beam-columns. Vinnakota (1967, 1971, 1974) developed a spread of plasticity model to study regular and irregular steel frames up to the point of instability, where the individual members were connected rigidly to the joints or through pin connections. The nonproportional loading considered consisted of both joint loads and arbitrarily distributed transverse loads on beams and columns. The method of transfer matrices (Vinnakota 1967; Vinnakota and Badoux 1971), generalized to include the effect of intermediate plastic hinges, was used to calculate the stiffness matrix elements of partially plastified members. The displacement method was used to determine the unknown deformations (three per joint). Researchers (Meek and Lin 1990; Kitipornchai et al. 1988, 1990; Al-Bermani and Kitipornchai 1990, 1992; Chan et al. 1991; Chan and Kitipornchai 1988; Chan 1989; Ziemian 1990; Vogel 1985) have used the spread of plasticity method in their analysis of beam columns and FR frames. The full nonlinear load-deformation response, including the unloading path, was computed for the case of small structures.

Ackroyd (1979), Ackroyd and Gerstle (1982), Cook (1983), Cook and Gerstle (1987) studied the inelastic behavior of PR frames. The unloading portion of the load-deformation response was not computed. Further, these studies concentrated on implementation of a modified plastic zone approach, using the column analogy to generate gradual yielding models that did not consider the shift in the neutral axis of the elastic core after cross-sectional yielding begins. Deierlein et al. (1990, 1991) presented the results of PR frame analysis where concentrated plasticity models were used. To the writers' knowledge, there have been no research studies on implementation of the classical spread of plasticity model in the analysis of large-scale FR and PR steel frames where full nonlinear load-deformation response (including unloading) was computed and the shifting location of the elastic neutral axis was considered.

Modifications to the simple plastic-hinge method have been proposed as a result of its analytical limitations and the com-

[1]Asst. Prof., Dept. of Civ. and Envir. Engrg., Marquette Univ., Haggerty Engrg. Hall #253, P.O. Box 1881, Milwaukee, WI 53201-1881. E-mail: c.foley@marquette.edu

[2]Prof., Dept. of Civ. and Envir. Engrg., Marquette Univ., Haggerty Engrg. Hall #261, P.O. Box 1881, Milwaukee, WI. E-mail: vinnakotas@vms.csd.mu.edu

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putational expense incurred when applying the plastic-zone method. These modifications most often attempt to smooth the abrupt plastification that is assumed to occur in the simple plastic-hinge model, or allow concentrated hinges to form along the member length (King 1990, 1994; Abdel-Ghaffar 1992). Attalla et al. (1995) presented a quasi-plastic-hinge-based analytical method that simulates spread of plastification along the member and through the section using elastic and inelastic curvature relations along the length of a finite element. The modification proposed by Attalla et al. (1995) still requires moment-curvature thrust relations for cross-sectional shapes and yield surfaces prior to implementation of the method. Spread of plasticity analysis needs no additional information other than a cross-sectional discretization and residual stress pattern. Therefore, with computational technology in its current state, the spread of plasticity model can become the most direct method of inelastic analysis. Also, modifications or improvements to simple plastic-hinge analysis require calibration for frames of general configuration. This is most effectively accomplished using spread of plasticity models.

This paper seeks to expand the current state of knowledge in the inelastic response of planar PR and FR structural steel frames. Derivation of a finite element capable of modeling spread of plasticity within the cross section, a variety of residual stress patterns, and a shift in the neutral axis of the elastic core is described. A second objective of the paper is to present accurate solutions to inelastic stability problems where the entire load-deformation response is computed using a simple reliable algorithm based on constant work and simple Euler stepping. Further, the paper intends to illustrate the validity of the finite element derived and numerical solution algorithm implemented through comparison with published analytical work. Finally, differences in analytical results for PR frame analysis using plastic-hinge-based analytical models and the plastic-zone method are to be presented. A companion paper (Foley and Vinnakota 1999) illustrates use of the finite element and solution algorithm presented in the inelastic ultimate load analysis of large-scale frameworks likely to be found in practice. Evaluation of connection classification systems (Bjorhovde et al. 1990; Goto and Miyashita 1996) are conducted on large-scale frames subjected to a variety of loading sequences.

FINITE ELEMENT

The Rayleigh-Ritz method and principle of minimum potential energy is used in the development of the finite-element stiffness equations. The typical finite element used as the basis for the development of the present analytical model is shown in Fig. 1(a). Assuming elastic–perfectly plastic material behavior (unloading occurs with the initial modulus and strain hardening is neglected), the strain energy for a finite-element can be written as follows:

$$U = \int_{V_e} \int_{\varepsilon_y} \sigma \, d\varepsilon \, dV_e + \int_{V_p} \left(\int_0^{\varepsilon_y} \sigma \, d\varepsilon + \int_{\varepsilon_y}^{\varepsilon} \sigma_y \, d\varepsilon \right) dV_p \quad (1)$$

Normal (incremental) strain, with reference to the nonmoving x, y, z coordinate system shown in Fig. 1(b), may be written using superposition of axial and bending strains (assuming elastic behavior within a load increment)

$$\varepsilon = \frac{du}{dx} + \frac{1}{2}\left(\frac{dv}{dx}\right)^2 - y\frac{d^2v}{dx^2} \quad (2)$$

Using Hooke's Law for elastic fibers, substituting (2) into (1), and integrating over the area gives the strain energy for the partially plastified element as follows:

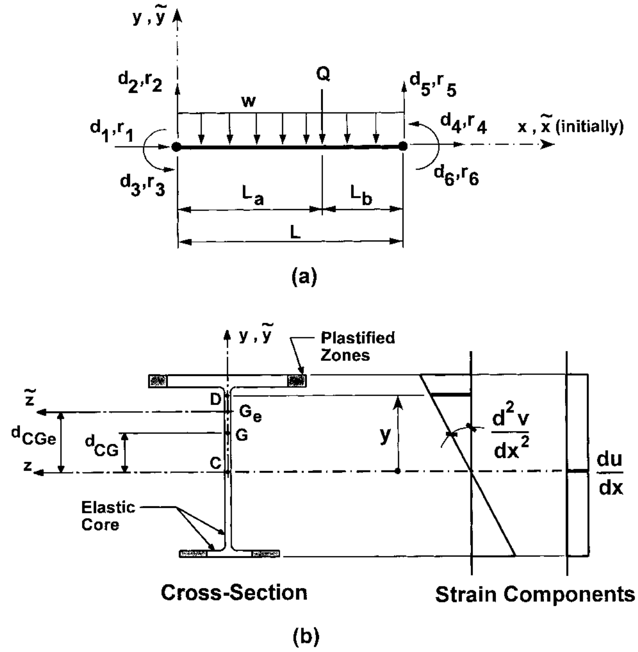

FIG. 1. Beam-Column and Coordinate System Used in Development of Finite-Element Equations: (a) Typical Finite Element with Loading; (b) Cross-Sectional Coordinate System with Plastified Zones

$$\begin{aligned} U = {} & \frac{E}{2}\int_0^L \left[A_e\left(\frac{du}{dx}\right)^2 - 2S_{ze}\left(\frac{d^2v}{dx^2}\right)\left(\frac{du}{dx}\right) + I_{ze}\left(\frac{d^2v}{dx^2}\right)^2 \right] dx \\ & + \frac{E}{2}\int_0^L \left[A_e\left(\frac{du}{dx}\right)\left(\frac{dv}{dx}\right)^2 - S_{ze}\left(\frac{d^2v}{dx^2}\right)\left(\frac{dv}{dx}\right)^2 + \frac{A_e}{4}\left(\frac{dv}{dx}\right)^4 \right] dx \\ & + \int_0^L \left[\int_{A_p} \sigma_y \left(\frac{du}{dx}\right) dA_p + \int_{A_p} \frac{1}{2}\sigma_y\left(\frac{dv}{dx}\right)^2 dA_p \right] dx \\ & - \int_0^L \left[\int_{A_p} \sigma_y y \left(\frac{d^2v}{dx^2}\right) dA_p + \frac{1}{2}\int_{A_p} \sigma_y \varepsilon_y \, dA_p \right] dx \end{aligned} \quad (3)$$

Eq. (3) contains many of the terms commonly found in the usual strain energy representations for elastic beam-column elements, such as those in Chen and Lui (1991). The first moment of area, second moment of area, and cross-sectional-area terms are based on the remaining elastic core with reference to the fixed coordinate system shown in Fig. 1(b). These terms are similar to those used by Vinnakota (1982). The nonmoving coordinate system allows for unsymmetric yielding within the cross section to be tracked and for the eccentricity of applied loads (whose point of application remains in the fixed system), with respect to the remaining elastic core to be included. The cross-sectional properties may be defined with the help of Fig. 1(b) and can be expressed as the following integrals;

$$A_e = \int_{A_e} dA_e, \quad S_{ze} = \int_{A_e} y \, dA_e = A_e \, d_{CG_e},$$

$$I_{ze} = \int_{A_e} y^2 \, dA_e, \quad A_p = \int_{A_p} dA_p$$

The generalized stress relations for the plastified areas (Vinnakota 1982; Foley 1996) may be written as

$$P_{A_p} = \int_{A_p} \sigma_y \, dA_p, \quad M_{A_p} = \int_{A_p} \sigma_y y \, dA_p$$

The total potential energy for the finite element with loads indicated in Fig. 1(a) can be written as follows:

$$\Pi = \frac{E}{2}\int_0^L \left[A_e\left(\frac{du}{dx}\right)^2 2 - 2S_{ze}\left(\frac{d^2v}{dx^2}\right)\left(\frac{du}{dx}\right) + I_{ze}\left(\frac{d^2v}{dx^2}\right)^2\right] dx$$

$$+ \frac{E}{2}\int_0^L \left[A_e\left(\frac{du}{dx}\right)\left(\frac{dv}{dx}\right)^2 - S_{ze}\left(\frac{d^2v}{dx^2}\right)\left(\frac{dv}{dx}\right)^2 + \frac{A_e}{4}\left(\frac{dv}{dx}\right)^4\right] dx$$

$$+ \int_0^L \left[P_{A_p}\left(\frac{du}{dx}\right) + \frac{P_{A_p}}{2}\left(\frac{dv}{dx}\right)^2 - M_{A_p}\left(\frac{d^2v}{dx^2}\right) - \frac{P_{A_p}}{2}\varepsilon_y\right] dx$$

$$+ \int_0^L w \cdot v\, dx + Q \cdot v(Q) - \{r\}^T\{d\} \tag{4}$$

The cross-sectional properties and generalized stress relations represented in (4) are numerically evaluated using a fiber-element model. Sixty-six fiber elements, as shown in Fig. 2(a), are used within the cross section of each finite element. In addition to assignment of initial stress states corresponding to a variety of residual stress patterns, this fiber configuration analytically allows for accurate modeling of cross-sectional yielding.

Linear shape functions for longitudinal displacement and cubic Hermite shape functions for transverse displacements with respect to the fixed coordinate system are used. All longitudinal, transverse, and rotational displacements in the element occur with respect to the fixed reference axes. Furthermore, all cross-sectional properties are evaluated with respect to these axes. Assuming linear (elastic) behavior within an incremental step, the longitudinal and transverse strains can be taken as the simple superposition given in Eq. (2). Normal strains corresponding to the location of the elastic centroid of the cross section are automatically adjusted for the curvature about the fixed z-axis in the finite-element equations (Foley 1996). The location of the elastic centroid within the cross section is therefore a moot point during analysis. All displacements, cross-sectional properties, generalized stress relations, etc., are evaluated with respect to the fixed coordinate system.

Fig. 2(b) illustrates the use of multiple finite elements to model each beam and column. As a result, P-δ effects and along-the-length plastification are automatically included in the analysis. The fact that the bowing effect has been neglected and that average curvatures are assumed over the finite-element length should be noted. And, therefore, a minimum number of finite elements along the length of each member to be used for each beam or column must be determined through calibration with previous analytical or experimental results.

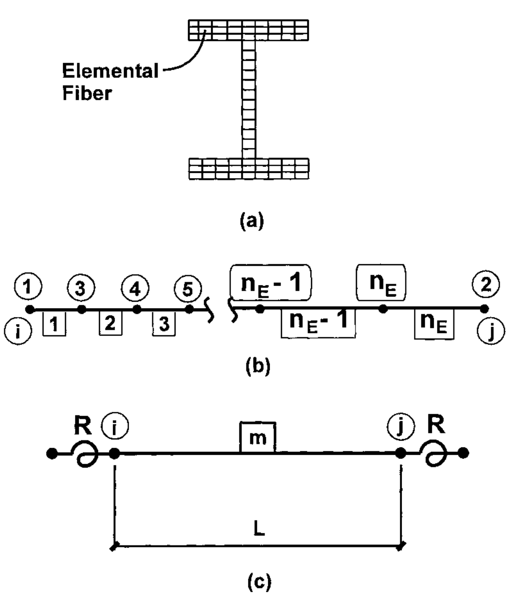

FIG. 2. Modeling of Beam and Beam-Column Elements: (a) Element Fiber Layout; (b) Subelements for Member Modeling; (c) Combined Finite-Element Model

The principle of stationary potential energy is used to obtain secant elemental stiffness relations given by

$$\{r\} = [K]\{d\} + \{\text{FEA}\} + \{r_P\} \tag{5a}$$

The element stiffness matrix is written as

$$[K] = [K_o] + \frac{EA_e}{2}[K_1] + \frac{EA_e}{3}[K_2] + [K_3] \tag{5b}$$

The tangent stiffness matrix terms follow from the secant relations. Elements found in the component matrices and details of the derivation are presented in Foley (1996). The vectors of fixed-end actions and plastification forces drop out of the incremental equations as a result of the partial differentiation, which implies that these quantities do not change over an incremental load step. Further, the cross-sectional properties, A_e, S_{ze}, I_{ze}, d_{CG_e}, M_{A_p}, and P_{A_p} are also assumed to remain constant over incremental load steps.

BEAM-COLUMN ANALYSIS

Implementing the finite element in an inelastic analysis of beam columns and frames requires a careful and systematic approach. Residual stress patterns are included in the finite-element implementation (Lehigh 1965; ECCS 1984). The minimum number of finite elements required to model each member must be determined through comparison (calibration) with previous analytical results. The finite element developed was used to trace the full nonlinear load-deformation response for several beam-columns with a variety of loading conditions and support configurations (Fig. 3). The cylindrical arc-length method with modified Newton-Raphson iterations (Crisfield 1991) was used in the nonlinear solution algorithm.

Analytical results for the cantilever column are given in Fig. 4 (El-Zanaty et al. 1980). The present study considers the residual stress pattern (ECCS 1984), which compares favorably

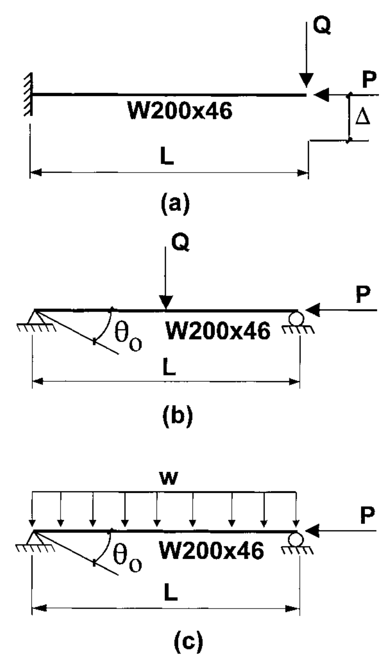

FIG. 3. Individual Beam-Column Configurations Used for Verification of Member Finite-Element Models with: (a) (El-Zanaty et al. 1980) Cantilever; (b) (Lu and Kamalvund 1968) Concentrated Load; (c) (Lu and Kamalvund 1968) Uniformly Distributed Load

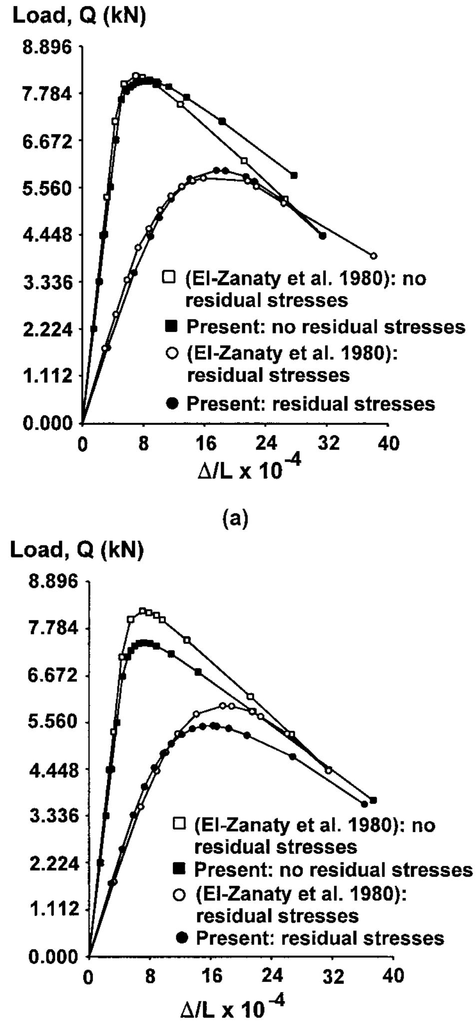

FIG. 4. Load-Deformation Response for (El-Zanaty et al. 1980) Column with $L/r = 14.4$ and $P/P_y = 0.91$: (a) Present, Three Finite Elements; (b) Present, 10 Finite Elements

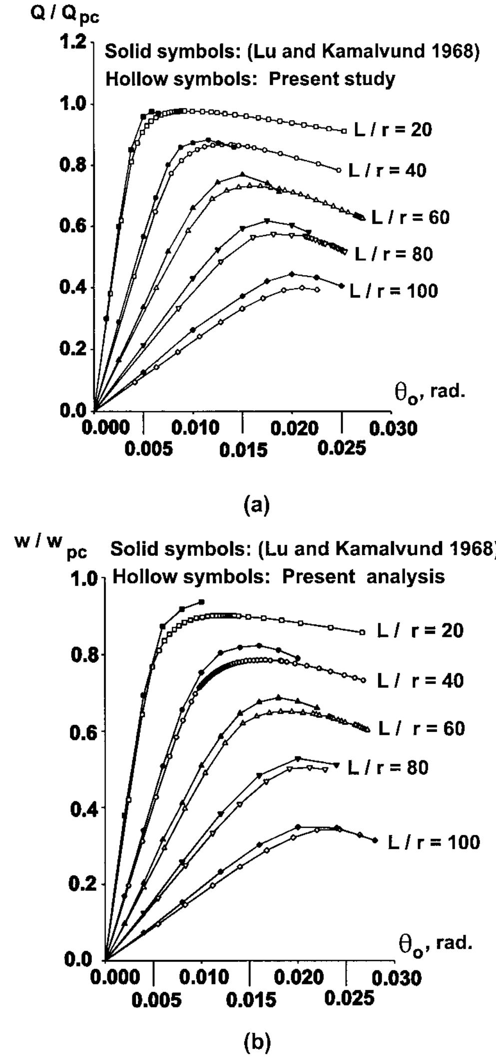

FIG. 5. Load-Deformation Behavior for (Lu and Kamalvund 1968) Columns with $P/P_y = 0.4$: (a) Concentrated Load; (b) Uniformly Distributed Loading

with the pattern used by El-Zanaty et al. (1980). Also, the ultimate load computed using 10 finite elements in the present formulation is considerably lower than in the results obtained by El-Zanaty et al. (1980). Although not completely clear, it is believed that El-Zanaty et al. (1980) used two finite elements (one being approximately 10% of the cantilever length at the fixed end, the other approximately 90% of the cantilever length) with multiple Gauss sampling points to aid in numerical integration. It is believed that the combination of slightly differing residual stress patterns, cross-section discretization and lack of a sufficient number of finite elements in El-Zanaty et al. (1980) are possible causes for the differences. It should also be noted that the performance of the arc-length algorithm can be seen in the symbol locations along the nonlinear response.

The simply supported columns studied by Lu and Kamalvund (1968) were also used for calibration of the finite element. Comparisons are given in Fig. 5(a) and 5(b). The present analysis considers residual stresses (Lehigh 1965). Excellent results are given for a wide variety of slenderness ratios for both loading configurations. Furthermore, full nonlinear load-deformation response for the beam-columns are provided using the present finite elements (10 total) and the arc-length method (with iteration). Lu and Kamalvund (1968) utilized moment-curvature-thrust relations derived assuming residual stresses (Lehigh 1965). The differences in the response are due to a slightly different residual stress pattern representation and slight numerical differences in cross-sectional representation.

FRAMEWORK ANALYSIS

Partial end restraint is introduced into the finite element using an established procedure (Lui and Chen 1986; Cook 1983). Each beam and column member within a framework must be modeled using a minimum number of finite elements, as described in the previous calibration. Of course, the analyst (engineer) could always use more than 10 elements. Thus, the degrees of freedom associated with nodes 3 through n_E in Fig. 2(b) are condensed, leaving six degrees of freedom associated with the structure joints (nodes i and j at the beam ends). A combined element is then formed for beam members using the aforementioned condensed element stiffness equations and two zero-length nonlinear spring elements attached to the ends of the beam members [as shown schematically in Fig. 2(c)]. An eight-degrees-of-freedom beam element is then formed and static condensation is used to remove unwanted degrees of freedom, resulting in a six-degrees-of-freedom combined-beam finite element. This combined element can then be assembled with column members in the usual manner.

Partially restrained connection models in this study utilize a multilinear moment-rotation curve. This model is simpler to implement in the computer program as the moment-rotation curve has consistent stiffness branches to help prevent numer-

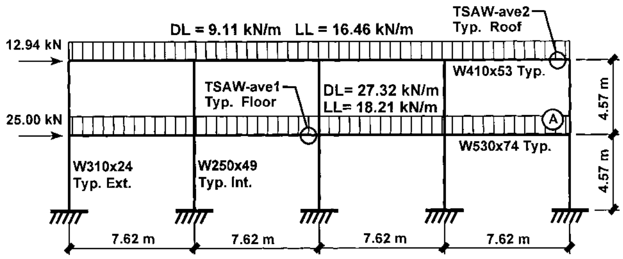

FIG. 6. Multistory, Multibay Structural Steel Frame Analyzed and Used for Verification in Present Study

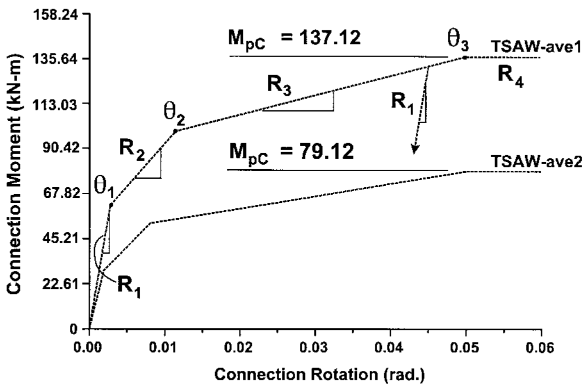

FIG. 7. Quadrilinear Connection Models Used in Present Study for Partially Restrained Frame Analysis Based on TSAW-ave Connection

TABLE 1. Parameters for Linearized Connection Method (Fig. 7)

Beam (1)	(kN-m/rad) R_1 (2)	R_2 (3)	R_3 (4)	$\theta_i(\times 10^{-3})$ (rad) θ_1 (5)	θ_2 (6)	θ_3 (7)	M_{pC}/M_{pb} (8)
W410 × 53	15,071	3,853	624	1.95	8.05	50.00	0.30
W530 × 74	22,189	4,267	987	2.80	11.44	50.00	0.30

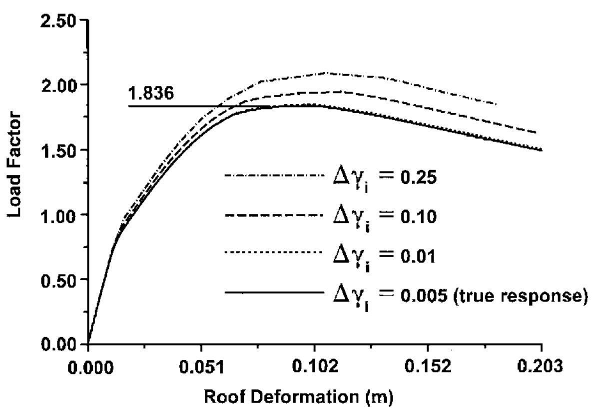

FIG. 8. Illustration of Numerical Algorithm Convergence to "True Response" for Partially Restrained Frame Configuration and (1.2DL + 0.5LL + 1.3WL)γ

ical instability. It should be noted that the stiffness of a spring element is assumed to remain constant during incremental load steps in the nonlinear algorithm.

There are numerous techniques to solve the nonlinear equilibrium equations that result from the present derivation. Powell and Simons (1981) and Yang (1984) developed techniques that involve constant work over each increment in load. The method used in this study to compute the complete nonlinear load-deformation path, including the ultimate load and post-critical response, is that proposed by Yang (1984). In the method, the simple Euler stepping algorithm is used in conjunction with constant work-load increments. Accuracy of the solution is controlled via multiple analyses based on convergence of system response. This algorithm is simple, reliable and is not sensitive to convergence failures that can occur in incremental-iterative schemes.

Use of the proposed algorithm (Yang 1984) rather than the arc-length method with equilibrium iterations used in the beam-column analysis was motivated by convergence failures in several frame runs. The beam-column analysis utilized displacement tolerance to establish convergence. It was difficult to consistently attain convergence of the arc-length method (with modified N–R iterations) on framework problems. Further, use of displacement tolerance as the only criteria for convergence tended to cause difficulty in the algorithm resulting from the relative magnitudes of lateral (frame) sway displacements when compared to nodal rotations. Convergence failures near the ultimate load of a highly redundant structure is discussed in detail by Crisfield (1991). Finally, when iteration is omitted, surpassing of limit points in the nonlinear response is consistently handled.

There are few convergence issues related to simple incrementation with constant work control. Thus the analyst does not have to suffer through (in some cases) many instances of convergence failure during incremental-iterative analysis to capture full nonlinear load-deformation response. Also, there can be problems that require manipulation of the arc-length procedure when general structures are analyzed. Certain structures behave well under displacement control, while others behave well under a blending of load and displacement control. It is difficult to judge the structure (especially with connection and material nonlinearity) prior to analysis. Furthermore, structure behavior can be significantly different when a vareity of load sequences are studied. Therefore, the reliability (not the accuracy) of incremental-iterative procedures can become questionable. Crisfield (1991) discusses means with which to manipulate the arc-length method in detail. In the case of simple stepping, the analysis will always give results that must be carefully monitored.

In lieu of converging within a load step as in incremental-iterative procedures, simple stepping with work control converges to the true response of the structure. An analyst can learn much more from a successful yet moderately accurate solution, than a solution that is truncated due to failed convergence within a load step. Further, Yang (1984) has demonstrated that simple stepping with work control can compete (in terms of accuracy) with incremental-iterative methods of analysis. Analysis of a PR frame will illustrate the accuracy that can be obtained using simple stepping with work control in elastic PR frame analysis.

The multistory, multibay structural steel frame analyzed (Deierlein et al. 1990) is used in this study to verify the accuracy of the computer program developed. The frame studied is shown in Fig. 6. The multilinear connection models for the TSAW-ave configuration [defined by Deierlein et al. (1990)] used in this study are shown in Fig. 7, and data are presented in Table 1. The present analysis uses 30 finite elements per beam, 20 finite elements per column, and a material that neglects strain hardening.

The convergence of several runs implementing the constant-work simple Euler stepping algorithm is shown in Fig. 8. Elapsed wall-clock time for the analysis with $\Delta\gamma_i = 0.005$ was approximately 25 min on a 200 MHz Pentium Pro personal computer. Although several runs are required to determine the load factor increments needed for acceptable accuracy, each run gives the full nonlinear load-deformation response. It can be seen that quite satisfactory results (<6% difference with respect to true response) can be obtained with an initial incremental load factor of 0.10 with no iteration at each load step.

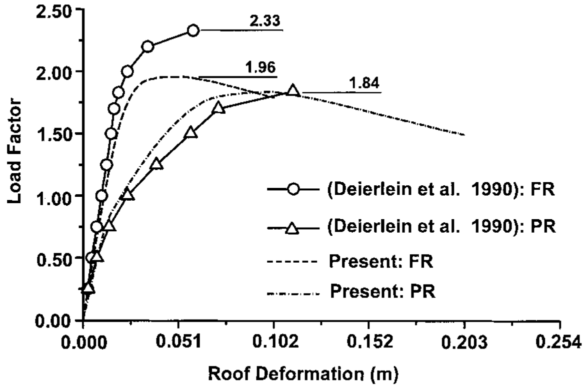

FIG. 9. Comparison of Load-Deformation Response for Multistory, Multibay Frame (Deieirlein et al. 1990) with TSAW-ave Connections: (1.2DL + 0.5LL + 1.3WL)γ

Other frameworks used for verification of the finite-element model and solution algorithm can be found in the literature (Foley 1996; Foley and Vinnakota 1997a).

The nonlinear load-deformation response for the structure with both FR and PR connections is given in Fig. 9. It should be noted that the residual stress pattern (ECCS 1984) and the full (nonreduced) yield stress are used in the present analysis. As can be seen, significant differences were obtained for ultimate load in the case of FR connections, while small differences were obtained for ultimate load in the case of PR connections. The analysis (Dierlein et al. 1990) used a plastic-hinge model with four finite elements per beam and two per column as well as a scaled yield surface. Partially-restrained connections tend to offset the increased analytical stiffness of the beams in the plastic-hinge model. Overall, the differences in ultimate load can be quantified and the analytical comparisons are favorable. The presence of residual stresses was found to alter the ultimate load by a negligible amount. The maximum elemental fiber strain in the PR framework computed in the present analysis occurred in the leewardmost column. This strain was 0.019 in./in. (compression). If one assumes grade A36 structural steel, strain hardening can be assumed to commence at a strain equal to approximately ε_{st} = 0.014 in./in. (Salmon and Johnson 1990). Therefore, strain hardening is probably occurring in the fibers of this column. However, the amount of strain beyond ε_{st} computed in this study is not expected to significantly alter the results of the analysis.

The spread of yielding as a percentage of initial cross-sectional area at the ultimate load condition is given in Fig. 10(a). It should be noted that a first-story sway mechanism appears, which is to be expected for this type of low-rise framework with moderate gravity load magnitude. Furthermore, plastic-hinge-type yielding is occurring in the columns (i.e., yielding is clustered at the column ends). However, yielding is progressing at a significant distance along the members. At the second-floor joints in the present analysis, rotational deformation is developed primarily through yielding at the first-floor column tops; in the case of the leewardmost column, deformation is developed at the first floor column top and second-floor column base. The resulting distributed yielding in the columns allows the connections within the model to be relieved of rotation when compared to the plastic-hinge-based analytical model. The maximum connection rotation at ultimate in the frame (Deierlein et al. 1990) was 0.042 rad (it is believed that this rotation corresponds to a second-floor connection located at an interior column). In the present analysis, the maximum connection rotation (second floor at an interior column) was computed as 0.014 rad. These results support the previous hypothesis related to column yielding and rotation demand.

An increased rotational demand on the beam connections at

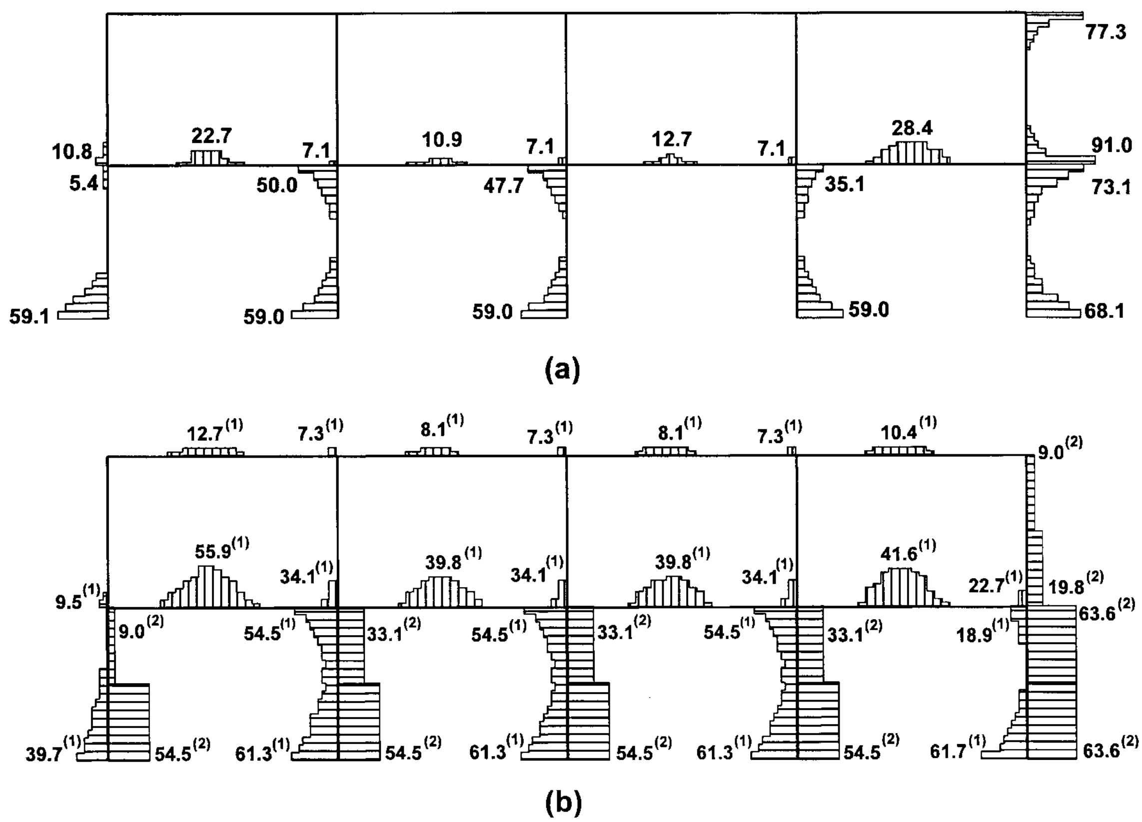

FIG. 10. Spread of Yielding as Percentage of Initial Cross-Sectional Area at Limit Load for the (Deierlein et al. 1990) Frame, (1.2DL + 0.5LL + 1.3WL)γ: (a) Original Frame Configuration (γ_u = 1.84); (b) Modified Frame: W200 × 71 Exterior Columns[(1)] (γ_u = 2.22) and Coarse Mesh[(2)] (γ_u = 2.07)

ultimate load can be illustrated using two approaches: (a) increasing the plastic moment capacity of the exterior columns; and (b) using a coarse finite-element mesh for the members (two elements per column and four per beam). To most easily illustrate this point, the leeward connection of the rightmost second-floor beam will be studied (connection A shown in Fig. 6). In the first case, the original W310 $\times$ 24 exterior columns are replaced by W200 $\times$ 71 columns. The mesh density remains unchanged (20 elements per column, 30 elements per beam). The connection rotation at ultimate and the ultimate load factor for this modified frame are 0.019 and 2.22 rad, respectively. There is a significant increase in ultimate load factor (20%) for this modified configuration due to the column cross-sectional increase. The spread of yielding within this modified frame is shown in Fig. 10(b). Less yielding in the exterior columns (most predominant at the top) now occurs; however, more yielding within the beam spans occurs as a result of the connections incurring larger rotations (due to more applied load on the beams). As a result of the interior columns remaining unchanged, yielding along the lengths of these columns at the higher-load levels is significant. The connection rotation at A in the original model (Fig. 6) was 0.0008 rads.

A second model using a relatively coarse finite-element mesh also illustrates the stiffening and increased connection A rotation demand. The coarse mesh models the ultimate load reasonably well (γ_u = 2.07), but the comparison of yielding spread shown in Fig. 10(b) with the original frame shown in Fig. 10(a) illustrates its expected deficiency. The leeward-most second-story column in the revised frame configuration undergoes much less yielding. This leads to an analytically stiff member and an increase in connection rotation demand at connection A (0.006 rad). There is only 12% more loading applied to the modified framework, yet there is a significant increase in rotational demand at connection A.

The present analysis indicates that although plastic-hinge models may capture the ultimate load of a frame with acceptable accuracy, the increased member stiffness, due to the fact that partial plastification along the length is not included in these models, may cause overestimation of the connection rotations once yielding within the members commences. In many instances, critical factors related to frame performance do not concern collapse, but local buckling and connection performance. Plastic-hinge analytical models may not adequately address these issues without modification. The work of Attalla et al. (1995) appears to be a step in this direction and, therefore, improved analytical results for frameworks can be expected without resorting to plastic-zone analysis.

CONCLUDING REMARKS

A finite element for use in structural steel framework analysis has been developed. The element considers gradual plastification within the cross section, residual stresses, and shift in the elastic core as yielding commences. Gradual plastification along the member length is considered through use of multiple finite elements. The finite element developed is shown to have sufficient accuracy in the analysis of structural steel beam-columns. Further, accuracy of the finite element and numerical algorithms used in the analysis of fully and partially restrained steel frames has been demonstrated. Here, the spread of yielding, ultimate load factor, and connection rotation have been compared with those of previous studies. It has been shown that plastic-hinge-based analytical models may result in overestimation of connection rotation at ultimate load and also neglect the formation of significant along-the-length yielding in low-rise frameworks, thereby leading to overestimation of ultimate loads.

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APPENDIX II. NOTATION

The following symbols are used in this paper:

A = area of finite-element cross section;
b = breadth of element in wide flange section;
C = arbitrary point on finite-element cross section corresponding to support point or connection point to other members;
DL = vertical dead loads applied to floor framing of building frame;
d = depth of wide flange shape or distance locating pertinent reference point within finite-element cross section;
$\{d\}$ = vector of nodal displacements at ends of finite element;
E = modulus of elasticity of material comprising finite element;
{FEA} = vector of fixed-end actions that result from transverse loads applied between ends of finite element;
G = center of gravity;
H = overall height of structural steel frame;
I = second moment of area;
$[K]$ = stiffness matrix for finite element;
L = length of (or length along) finite element;
LL = vertical live loading applied to floor-framing members of building frame;
M = bending moment;
n = number of elements;
P = axial load applied to (or within) finite element;
Q = transverse concentrated load;
R = connection stiffness;
$\{r\}$ = vector of nodal loads at ends of finite element;
S = first moment of area;
SL = vertical snow load applied to roof-framing members of frame;
t = thickness of element in wide flange section;
U = internal strain energy for finite element;
u = function describing longitudinal displacement along finite element;
V = volume of finite element;
v = function describing transverse displacement along length of finite element;
WL = lateral wind loading applied to building frame;
w = uniformly distributed transverse load applied between ends of finite element;
x = longitudinal distance along length of finite element;
y = transverse distance to point on finite-element cross section from reference z-axis (also vertical cross-sectional reference axis through element connection point, center of gravity of elastic core, and center of gravity of unstressed cross section);
$\tilde{y}$ = vertical cross-sectional reference axis through center of gravity of remaining elastic core of finite-element cross section;
z = transverse cross-sectional reference axis through member connection point C on unstressed cross section of finite element;
$\tilde{z}$ = transverse cross-sectional reference axis through center of gravity of remaining elastic core;
γ = load factor for loading applied to building frame;
Δ = lateral deflection of topmost story in building frame, lateral deformation of tip of cantilever column, or relative sway between ends of beam-column member;
$\Delta\gamma$ = incremental load factor used in constant work simple stepping numerical algorithm;
δ = lateral deformation of beam-column member relative to chord defined between its ends;
ε = normal strain at any point within finite-element cross section;
θ = rotation at end of laterally loaded beam-column, or connection rotation; and
σ = normal stress at any point within finite-element cross section.

Subscripts

A_p = plastified portions of cross-sectional area;
a = from end i to concentrated transverse load;
b = from end j to concentrated transverse load;
CG = point C on cross section to center of gravity of unstressed cross section;
CG_e = stationary axis origin (point C) to center of gravity of remaining elastic core;
E = finite element;
e = remaining elastic core;
f = cross-sectional flange;
i = initial;
o = end of laterally loaded beam column;
p = plastified portion of cross section;
p_b = plastic capacity of beam member;
pC = connection plastic capacity;
p_c = reduced plastic capacity with respect to applied loading;
r_c = compressive residual stress;
r_t = tensile residual stress;
st = strain hardening;
T = tangent stiffness;
u = ultimate;
w = cross-section web;
y = yield;
ze = remaining elastic core with reference to fixed z-axis; and
1, 2, 3 = stage(s) in quadrilinear connection model, or component(s) of stiffness matrix.