Design of Partially or Fully Composite Beams, with Ribbed Metal Deck, Using LRFD Specifications

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A Hindu saying states, "One picture is worth a thousand words (numbers)." With this optic in mind, the design aspects of steel-concrete composite beams using the recently adopted LRFD specifications[1] are analyzed critically and the significance of several parameters is brought out clearly. Charts are then constructed to facilitate the design of partially or fully composite beams using rolled-steel, wide-flange sections of A36 steel or A572 Gr. 50 steel. The slab may be a composite metal deck slab with ribs perpendicular to the beam, a haunched slab or, simply, a flat soffit concrete slab. The charts cover both adequate and inadequate slabs. The charts provided are a valuable tool from the practical standpoint, and also familiarity with them should contribute to the student's and young engineer's overall feel of the composite beam design problem. The design charts given here complement the composite beam design tables provided in the LRFD manual.

Introduction

A typical bay floor framing of a high-rise building consists of steel floor beams framing into steel girders along bay lines (Fig. 1). The floor beams generally are designed for only gravity loads imposed by the floor as simply supported composite beams with the slab essentially in compression over the full span of the beam. The beam normally is of a standard rolled, wide-flange shape designed to interact compositely with the concrete floor slab by means of shear studs placed in metal decking troughs. In such a metal deck composite beam system, the ribs or corrugations generally run perpendicular to the supporting floor beams.

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The *Load and Resistance Factor Design Specification (LRFD) for Structural Steel Buildings*[1] adopted by AISC in September 1986 uses the ultimate strength of composite beams as the basis of their design. According to LRFD, composite beam designs are classified as fully composite and partially composite designs.[2,3,4,5,6,7,8,9,10] The present report on the design of partially composite beam design is a generalization of the work presented earlier in Ref. 11.

COMPOSITE METAL DECK SLABS

Composite metal deck slabs consist of light-gage, ribbed metal deck forms which interact with structural concrete topping as a composite unit to resist floor loads (Fig. 2). Special embossments, dimples or lugs cold-rolled into the decking increase bond and act as shear connectors. Uplift is prevented either by the shape of the profile or by inclining the lugs to the vertical in opposite directions, on the two sides of the rib. It is usual practice to design the slab as a one-way, simply supported beam, for the ultimate limit state (with the metal decking acting as reinforcement steel in the span direction), even though the slab and the decking may be continuous over the floor beams. The slab is usually provided with square mesh steel reinforcement at, or above, mid depth of the slab to minimize cracking due to shrinkage and temperature effects and to help distribute concentrated loads.

The variables for a composite deck slab include span length, gage thickness, rib depth, slab thickness, unit weight of concrete and concrete strength.[12,13,14] Thickness of metal deck plate elements usually varies from 22 ga. (0.0336 in.) to 12 ga. (0.1084 in.), depending on configuration of the section. For noncellular decks 1½ in., 2 in. and 3 in. deep decks are generally used for spans up to 8 ft, 10 ft and 15 ft, respectively. The thickness of concrete above the metal deck typically varies from a minimum of 2½ in. to 4 in., which may be controlled by fire-rating requirements of the slab instead of structural requirements. The choice of lightweight concrete or normal weight concrete, depends on economics and fire-rating considerations. For high-rise buildings, light-

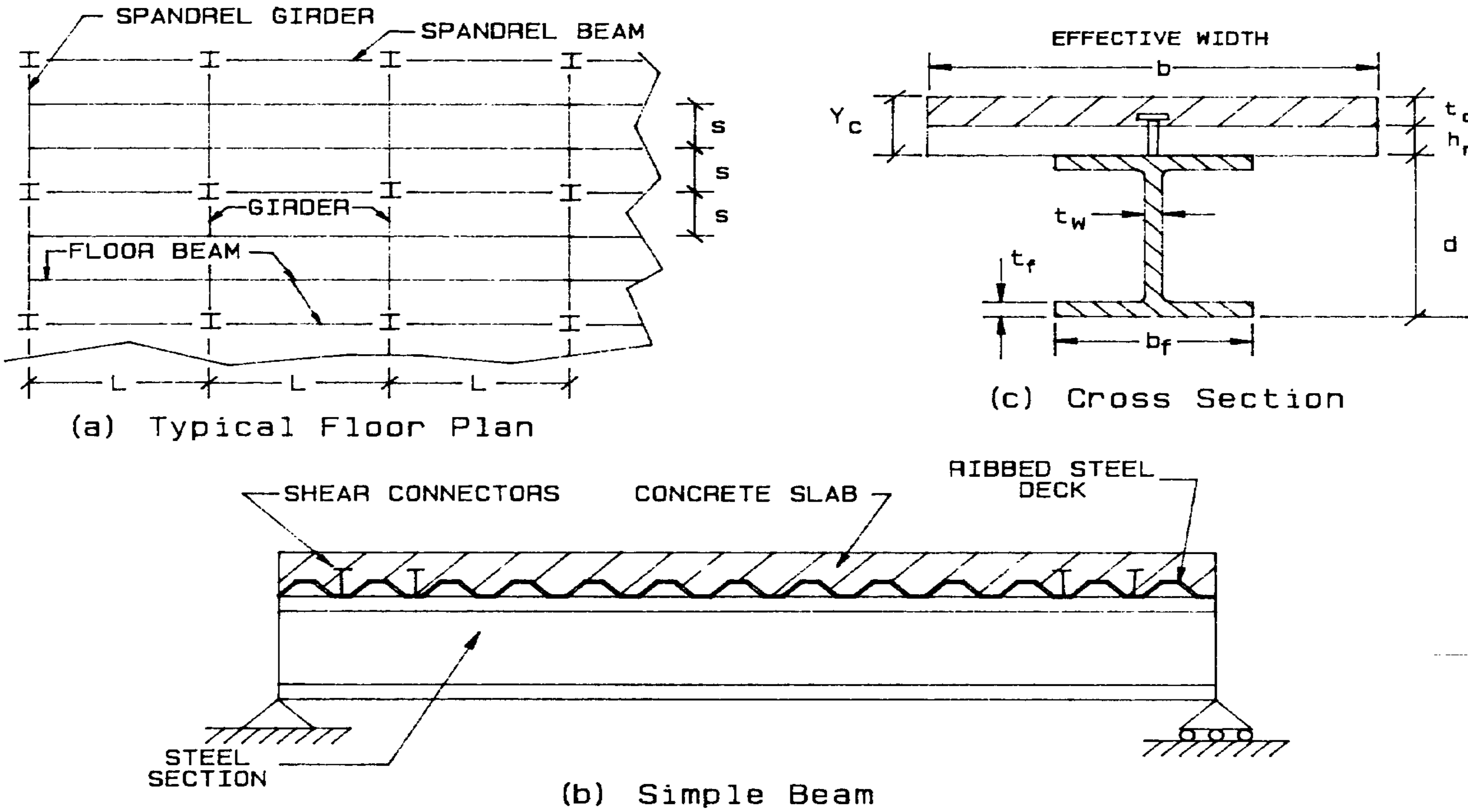


Fig. 1. Composite beam under study

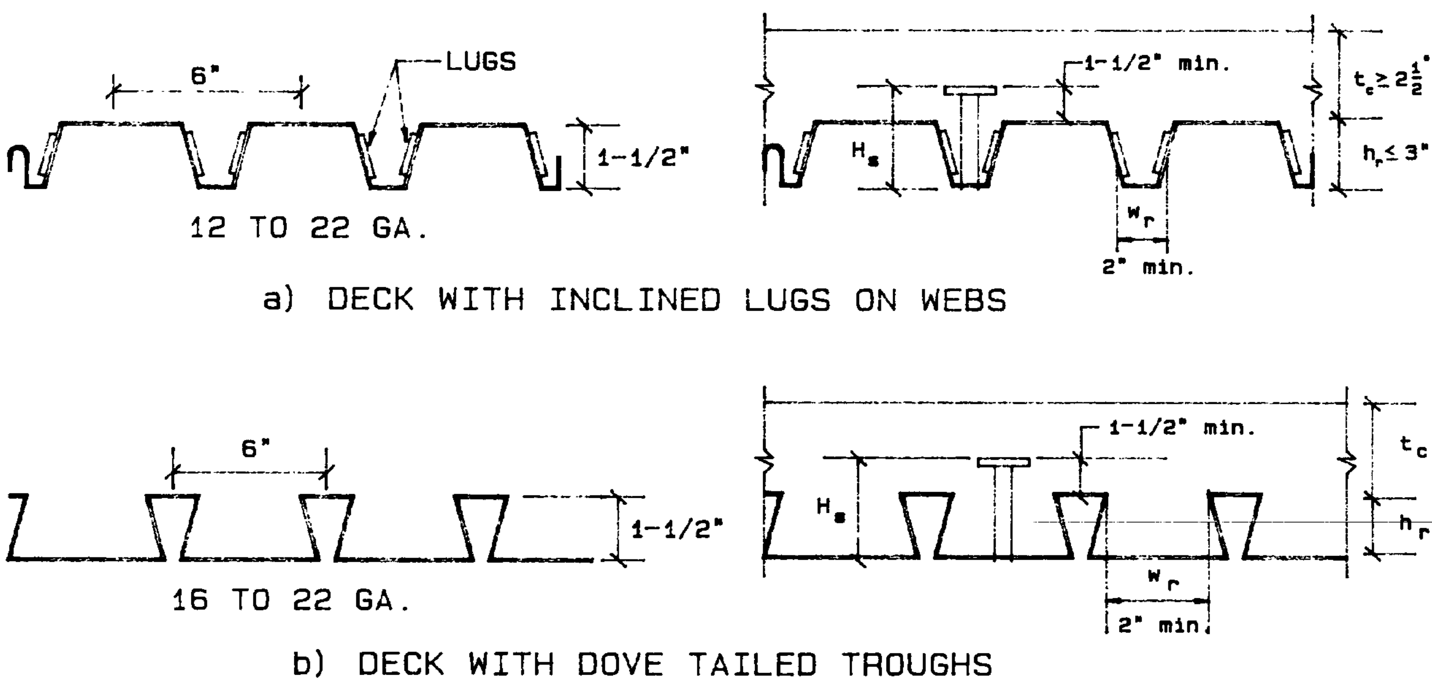


Fig. 2. Composite metal decks

weight concrete weighing about 110 pcf is often used.

The advantage of the metal deck slab system is the elimination of the formwork and shoring and the consequent increase in speed of construction. The metal deck can be used as a working and erection platform. Also, it acts as a diaphragm to help stabilize the steel skeleton by integrating all members into a system.

The steel deck alone has to withstand the weight of the wet concrete, plus any construction loads for placement of concrete, for the desired non-shored condition. The depth of the ribs necessary is generally controlled by this loading. The composite steel deck (steel deck + concrete) has to withstand the factored superimposed dead and live loads. Design of the steel deck should conform to the latest edition of the specifications for the design of light gage, cold formed steel structural members of the American Iron and Steel Institute (AISI). The permissible superimposed floor loads are tabulated for different deck sections by their manufacturers. The tabulated capacity is obtained as the smallest load of the following conditions: 1. bending stress at the bottom fiber of the deck; 2. bending stress in the top fiber of the concrete; 3. bending stress at the top fiber of deck under deadload only; 4. value of the transverse shear for shear bond failure and; 5. deflection at mid-span. Hence, a detailed design is not performed by the structural engineer.

SHEAR CONNECTORS

The purpose of shear connectors in a composite beam is to tie the slab and steel beam together and force them to act as a unit. For this, the connectors must resist the horizontal shear force that develops between the slab and beam as the composite member is loaded, and they should prevent vertical separation or uplift of the concrete slab from the steel beam.

Presently, stud connectors are the most commonly used shear connectors in the U.S. The stud shear connector is a short length of round steel bar, welded to the steel beam at one end, with an upset end or head at the other end. They range in diameter from ½ in. to 1 in. and lengths from 2 to 8 in. The ratio of the overall length to the diameter of the stud is not less than 4. The most commonly used sizes in building structures are ¾ in. or 7/8 in. dia. The head diameter is ½ in. larger than stud diameter and the head thickness is 3/8 in. or ½ in. The anchorage provided by the head on the stud ensures the required uplift resistance. The studs are made with ASTM-A108, AISI Grades C1010, C1015, C1017 or C1020 cold-drawn steel with a minimum tensile strength of 60 ksi and a minimum elongation of 20% in 2-in. gage length, as specified in the AWS Structural Welding Code D1.1-75.

To prevent premature failure of studs because of tearing of base metal, the size of a stud not located over the beam web is limited to 2½ times the flange thickness.[1] The strength of stud connectors increases with stud length up to a length of about four diameters and remains approximately constant for greater lengths. For design purposes, the connector strengths are always expressed in terms of an equivalent shear load. Thus, according to LRFD specifications, the nominal strength of one stud shear connector embedded in a solid concrete slab is

$$Q_n = 0.5A_{sc}(f'_c\,w)^{3/4} \quad (1a)$$

$$\leq A_{sc}\,F_u \quad (1b)$$

where

A_{sc} = cross-sectional area of a stud shear connector, in.^{2}
f'_c = specified compressive strength of concrete, ksi
w = unit weight of concrete, pcf
F_u = minimum specified tensile strength of connector steel, ksi

Table 12 of the LRFD specification gives nominal shear loads Q_n for the standard range of welded stud shear connectors for normal weight (145 pcf) concrete made with ASTM C33 aggregates. For lightweight (90 to 110 pcf) concrete made with C330 aggregates, the values from Table 12 are to be adjusted by multiplying them with coefficients given in Table 13 of the LRFD Specification.

When metal deck is used, the studs are generally welded through it. When the ribs are relatively deep, the composite interaction between the slab and the steel beam is affected adversely by the reduced concrete containment around the roots of the connectors. The nominal strength of a stud connector for deck ribs oriented perpendicular to the beam is the value given by Eq. 1a, multiplied by the following reduction factor:[15]

$$R = \frac{0.85}{\sqrt{N_r}}\left(\frac{w_r}{h_r}\right)\left[\frac{H_s}{h_r} - 1.0\right] \leq 1.0 \quad (2)$$

where

h_r = nominal rib height, in.
H_s = length of stud connector, in., not to exceed the value (h_r + 3) in calculations, although actual length may be greater
N_r = number of stud connectors in one rib at a beam intersection, not to exceed 3 in calculations, although more than 3 may be installed
w_r = average width of deck rib, in.

The factor $0.85/\sqrt{N_r}$ accounts for the reduced capacity of multiple connectors, including the effect of spacing. Thus, the strength of a stud shear connector in a deck rib is given by

$$Q_{nr} = R\left[0.5A_{sc}(f'_c w)^{3/4}\right] \quad (3a)$$

$$\leq A_{sc}F_u \quad (3b)$$

As the load on a composite beam is increased, the heavily stressed connectors near the supports will begin to yield and they will deform without taking additional shear Hence, further loading will be carried by the lightly loaded inner connectors until eventually all the connectors are stressed to the yield point. Thus the connector flexibility permits a redistribution of forces so all the connectors between the points of maximum and zero bending moments become loaded equally. Therefore, it follows the exact spacing of the connectors is of little importance. In statically loaded structures, like the ones under consideration in the present paper, the connectors may therefore be spaced equally along the length of the beam.

The minimum spacing of connectors along the length of the beam in flat soffit concrete slabs is six diameters. When the ribs of the metal deck are perpendicular to the beam, the longitudinal spacing of shear connectors must of course be compatible with the pitch of the ribs. Also, since most test data are based on the minimum transverse spacing of four diameters, this transverse spacing was set as the minimum permitted. To control uplift and to avoid too irregular a flow of shear into the concrete, limits are placed on the maximum spacing of connectors along a beam. According to the LRFD Specification, the maximum longitudinal spacing of shear connectors shall not exceed 32 in. or eight times the total slab thickness.

FULLY COMPOSITE AND PARTIALLY COMPOSITE BEAMS

Figure 3a shows a simple-span composite beam with uniformly distributed load. If the magnitude of these loads is increased monotanically, the ultimate bending moment of the composite beam is reached at mid-span. Figure 3b is an exploded diagrammatic elevation of the composite beam showing the longitudinal forces acting on the concrete slab, between the mid-span and the end of the slab, as a free body separated from the steel beam. Let C equal the compressive force in the slab at mid-span, T equals the tensile force in the steel beam and H equals the horizontal shear force to be transferred from the slab to the steel section over the length $L/2$. From the longitudinal equilibrium of the slab,

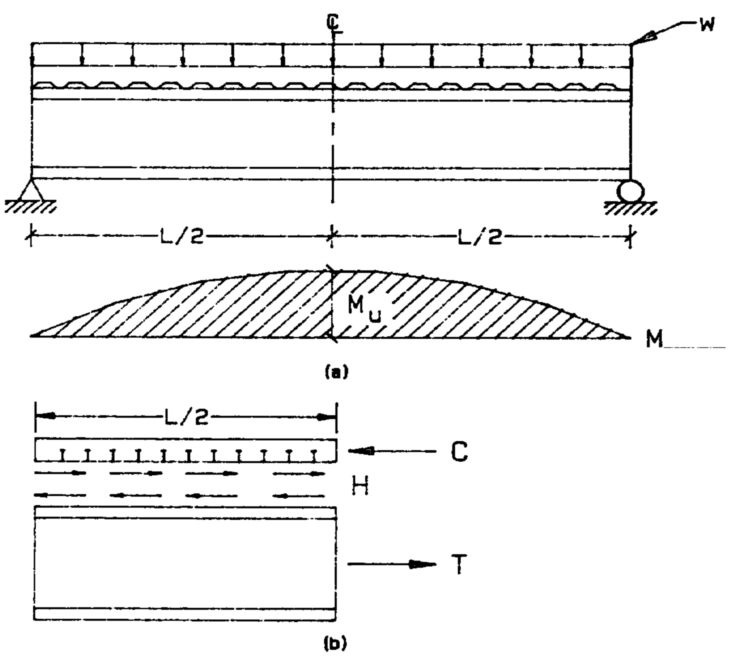


Fig. 3. Shear forces at ultimate moment

$$H = C \tag{4}$$

As natural bond and friction between the slab and the steel section are not relied upon, the horizontal shear resistance is to be provided by the shear strength of the connectors over the length $L/2$.

Let C^* represent the maximum compressive strength of the concrete slab, T^* the maximum tensile strength of the steel section and S the shear strength of the connectors between the point of maximum moment and the support point. Then,

$$C^* = F_c A_c = 0.85 f_c' b t_c \tag{5a}$$

$$T^* = F_y A_s \tag{5b}$$

$$S = N Q_{nr} \tag{5c}$$

where

A_c = area of slab based on the actual slab thickness t_c, in.^{2}
A_s = area of steel section, in.^{2}
b = effective width of the slab, in.
F_c = equivalent yield stress of concrete in compression, assumed to be equal to 0.85 f_c', ksi
F_y = yield stress of steel, ksi
f_c' = compressive strength (28-day cylinder strength) of concrete, ksi
N = number of connectors over half-span length
Q_{nr} = shear capacity of one connector, kips

The slab is said to be adequate when $C^* \geq T^*$ and inadequate when $C^* \leq T^*$. According to the LRFD Manual, the compressive force C in the concrete slab is the smallest of C^*, T^* and S. That is,

$$C = \min [C^*, T^*, S] \tag{6}$$

Introduce the notations S^*, N^* so that

$$S^* = \min [C^*, T^*] \tag{7}$$

$$N^* = S^*/Q_{nr} \tag{8}$$

Equation 6 can now be rewritten as

$$C = \min [S^*, S] \tag{9}$$

If the strength of the shear connectors actually provided S is less than S^*, the beam is said to be a *"partially composite beam."* On the other hand, if the strength of the shear connectors actually provided S is greater than or equal to S^*, the beam is said to be a *"fully composite beam."* Thus,

$$S < S^* \text{ and } N < N^* \text{ for partially composite design}$$
$$S > S^* \text{ and } N > N^* \text{ for fully composite design} \quad (10)$$

A composite beam in which the concrete, steel and connectors are all of the same strength is said to be of a *balanced design*. Thus, for a balanced design,

$$C = C^* = T^* = S^* = S \quad (11)$$

The thickness of the slab and the size of the steel beam are often determined, in practice, from factors other than their strength when acting together as a composite beam; and its strength, when calculated by full interaction theory, is found to be greater than required for actual loading. The most economical design may then be one in which the number of shear connectors provided in a half-span N is such the degree of interaction between the slab and the steel section is just sufficient to provide the required flexural strength, and is less than the number N^* required for a fully composite design. Some such situations for use of partial composite action are:

1. An oversized steel beam must be selected from the available rolled beam sizes for architectural reasons or ease of fabrication (repeatability) or when deflection controls and strength requirements are adequately met by less than fully composite action.
2. When the ribs of the metal deck are perpendicular to the beam, the longitudinal spacing of shear connectors must be compatible with the pitch of the ribs. Quite often it may not be possible to fit in sufficient number of shear connectors, for the beam to be designed as a fully composite beam.

Decreasing the number of connectors for partial composite action reduces the effective stiffness of the composite beam slightly, so deflections may increase. The LRFD Manual recommends a lower limit for the connector strength S from practical considerations; namely

$$S_{\min} = 0.25\, T^* = 0.25\, A_s F_y \quad (12)$$

NOMINAL PLASTIC MOMENT OF COMPOSITE SECTIONS

According to the LRFD Specification, the moment capacity is approximated by the plastic moment of the composite section M_n, provided web dimensions satisfy local buckling criteria:

$$h_c / t_w \leq 640 \big/ \sqrt{F_y} \quad (13)$$

Here, t_w is the thickness of the web and h_c the web depth for stability. From Sect. B5.1 of the LRFD, h_c is twice the distance from the neutral axis to the inside face of the compression flange less the fillet. For simple beams considered in this report, h_c is a maximum when the concrete is poured and the composite action is not yet effective. For positive bending, rolled steel sections and simple beams considered in this report, $h_{c\ max}$ is therefore equal to the web depth clear of fillets (= $d - 2k$). For steels with F_y = 36 ksi and 50 ksi, to which the present report is limited, all the rolled sections tabulated in the LRFD Manual satisfy the criteria given by Eq. 13. So the design of composite beams considered could be based on the plastic moment M_n. This plastic moment is obtained by assuming the steel section is fully yielded (in tension or compression) and the compressed part of the concrete slab is everywhere stressed to 0.85 f_c', considered as an "equivalent yield stress for concrete." The tensile part of the concrete slab, if any, is assumed ineffective.

The equations for the plastic moment of a composite beam depend on the location of the plastic neutral axis (PNA), which in turn is determined by the relative proportions of the steel concrete and connector areas.[1,16,17] There are three fully plastic stress distributions to be considered:

Case a: applies when the plastic neutral axis is in the slab
Case b: applies when the plastic neutral axis is in the flange of steel section
Case c: applies when the plastic neutral axis is in the web of steel section

The assumed stress distributions are shown in Figs. 4, 5 and 6, respectively. The compression force C in the concrete slab, in these figures, is defined by Eq. 6. The depth of the compressive stress block in the slab is given by

$$a = \frac{C}{F_c b} = \frac{C}{085 f_c' b} \quad (14)$$

Let Y_c be the distance from the top of the steel beam to top of concrete and Y_2 the distance from top of steel beam to the compression force C in concrete. Then,

$$\begin{aligned} Y_c &= h_r + t_c && \text{for a metal deck slab} \\ &= h_h + t && \text{for a haunched slab} \\ &= t && \text{for a flat soffit slab} \end{aligned} \quad (15a)$$

$$Y_2 = Y_c - \frac{a}{2} \quad (15b)$$

where

h_r = height of rib, in.

h_h = height of haunch, in.

t = thickness of a flat soffit slab, in.

t_c = thickness of concrete slab above metal deck, in.

The dimension Y_2 is used as a parameter in the preparation of composite beam tables in the LRFD Manual and in the preparation of composite beam design charts

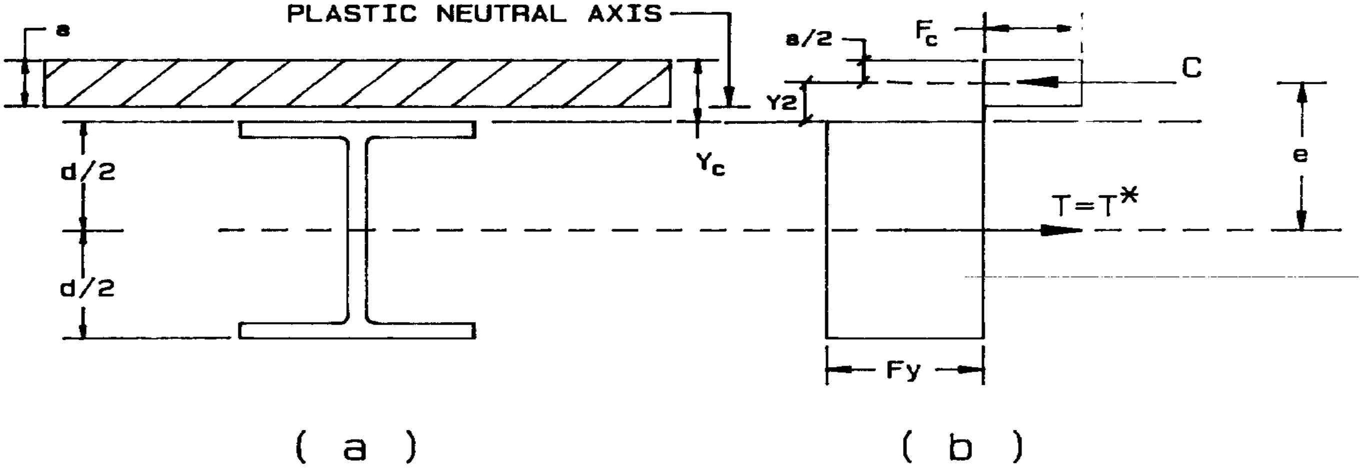


Fig. 4. Plastic neutral axis in slab (Case a)

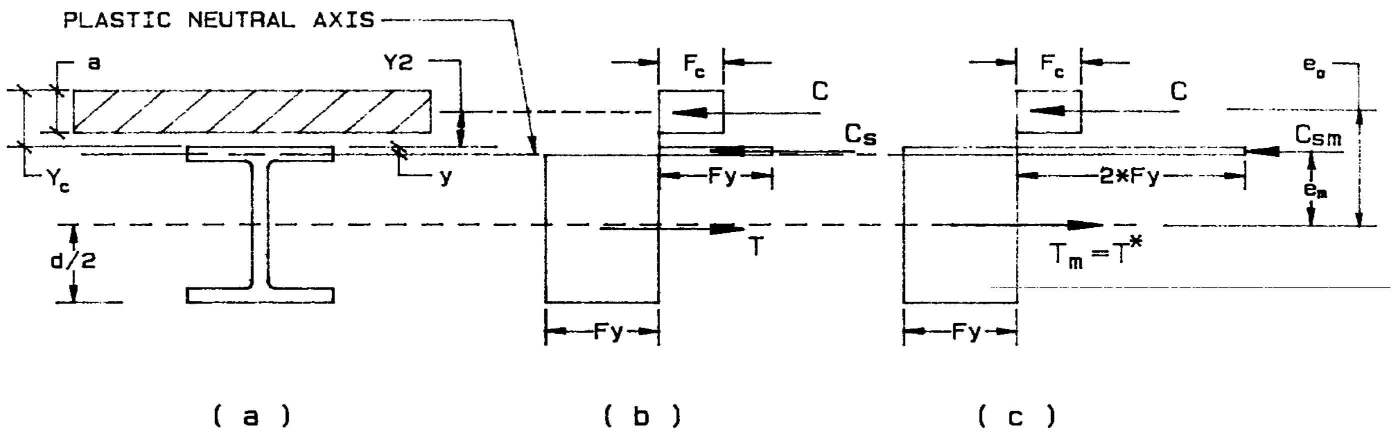


Fig. 5. Plastic neutral axis in flange (Case b)

developed in this paper.

Plastic Neutral Axis in Slab-Case a

When the PNA lies in the slab, the assumed stress distribution at flexural failure is shown in Fig. 4. The resultant compressive force in the slab is

$$C = F_c\, ba = 0.85\ f_c^{'}\ ba \qquad (16)$$

and it acts at a distance $a/2$ from the top of the slab. The resultant tensile force in the section is

$$T = F_y\, A_s = T^* \qquad (17)$$

where A_s is the area of the steel section and T^* is the yield force of the steel section. The force T acts at the middepth of the steel section.

For the longitudinal equilibrium of the composite beam,

$$C = T = T^* \qquad (18)$$

The plastic moment of the composite section is obtained by taking moments about the mid-depth of the steel section as

$$M_n = C\,e = T\,e = T^*\left[\frac{d}{2} = Y_2\right]$$
$$= 0.5\,T^*\,d + T^*\,Y_2 \qquad (19)$$

where e is the moment arm, d is the depth of the steel section and Y_2 is the parameter defined by Eq. 15b.

The PNA will be in the concrete, if

$$a \leq t_c \qquad (20)$$

By multiplying both sides by $0.85\ f_c^{'}b$ and using Eqs. 5, 16 and 17, this condition for the PNA to be in the concrete slab can be written as

$$C = T^* \leq C^* \qquad (21)$$

We observe that the steel section is the weaker of the two elements of the cross-section, namely, concrete slab and steel

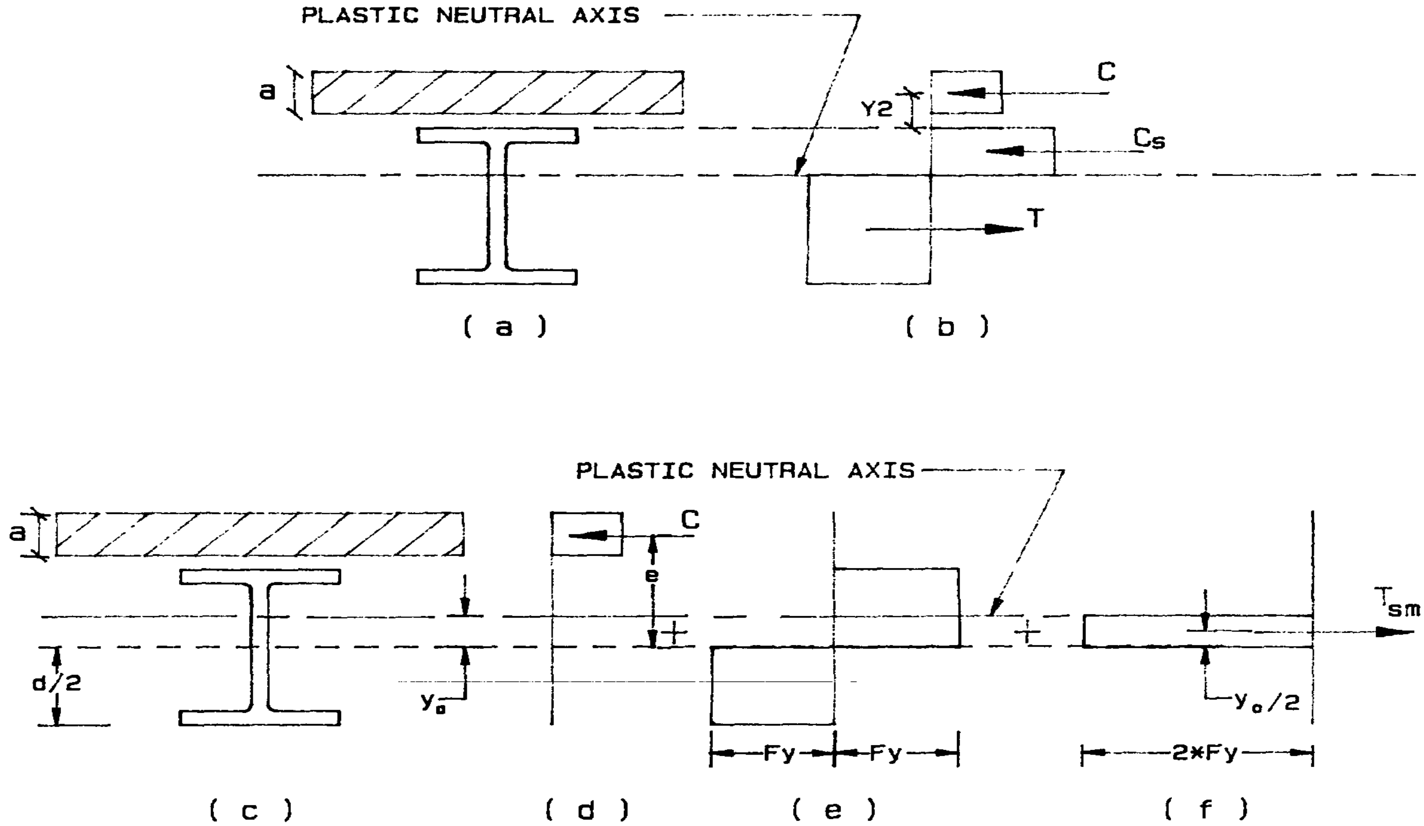


Fig. 6. Plastic neutral axis in web (Case c)

section, and the connectors have to develop the full strength of the steel section. The beam is therefore a fully composite beam, and the slab is adequate.

Plastic Neutral Axis in Flange-Case b

If the PNA is in the steel section, it yields in compression above the PNA and fully yields in tension below the PNA, as in Figs. 5a and b. Here C is the compressive stress resultant in concrete, C_s is the compressive stress resultant in the top flange and T is the tensile stress resultant in the steel section.

Although only the steel section below the PNA is in tension at yield stress, a simpler expression for the plastic moment M_n results by assuming that (by adding a pair of equal and opposite forces) the entire steel section is in tension and compensate for this by doubling the compressive yield stress in steel section above the PNA, as in Fig. 5c. The modified tensile force in the steel beam is

$$T_m = F_y A_s = T^* \qquad (22)$$

while the modified compressive force in the steel section is

$$C_{sm} = 2\, F_y\, A_{cs} \qquad (23)$$

where A_{cs} is the area of steel section at yield in compression. For longitudinal equilibrium,

$$T_m = C + C_{sm} \qquad (24)$$

which, using Eqs. 22 and 23, can be written as

$$T^* = C + 2\, F_y\, A_{cs} \qquad (25)$$

resulting in

$$C_{sm} = [T^* - C] \qquad (26)$$

and

$$A_{cs} = \frac{1}{2F_y}[T^* - C] \qquad (27)$$

Then, denoting by y, the distance from the top of the steel flange to the PNA,

$$y = \frac{A_{cs}}{b_f} = \frac{1}{2\, b_f\, F_y}\left[T^* - C\right] = \frac{(T^* - C)}{2\, P_{yf}}\, t_f \qquad (28)$$

where b_f and t_f are the width and thickness of the steel flange, respectively, and P_{yf} is the flange yield force. Therefore, for the moment arm,

$$e_m = \frac{d}{2} - \frac{y}{2} = \frac{1}{2}\left[d - \frac{(T^* - C)}{2\,P_{yf}}\, t_f \right] \qquad (29)$$

Taking moments about the mid-depth of the steel section,

$$\begin{aligned} M_n &= Ce + C_{sm} e_m \\ &= Ce + \frac{1}{2}(T^* - C)\left[d - \frac{(T^* - C)}{2\,P_{yf}}\, t_f \right] \\ &= C\left(\frac{d}{2} + Y_2\right) + \frac{1}{2}(T^* - C)\left[d - \frac{(T^* - C)}{2\,P_{yf}}\, t_f \right] \end{aligned} \qquad (30)$$

The PNA will remain in the flange if $0 \le y \le t_f$. From Eq. 28, these limits are seen to be $C = T^*$ and $C = T^* - 2\,P_{yf} = P_{yw}$ where P_{yw} is the web yield force. Thus, when $P_{yw} \le C \le T^*$, the PNA will be in the top flange. In addition, if $S < S^*$, the beam is a partially composite beam and the depth of the concrete compressive stress block a is less than the slab thickness t_c. On the other hand, if $S \ge S^*$, the beam is a fully composite beam and the entire slab is in compression.

Plastic Neutral Axis in Web-Case c

Figure 6b shows the plastic stress distribution where the PNA is in the web at a distance y_o above the mid-depth of the steel beam. This distribution can also be represented by three equivalent parts, as in Figs. 6d, e and f. It can be seen that the part in Fig. 6d represents the compressive force C in the concrete, acting at a distance e from the mid-depth of the steel section. The part in Fig. 6e represents the tension and compressive forces with zero stress resultant and a moment resultant equal to the plastic moment M_p of the steel section. And the part in Fig. 6f represents a tensile force in the web,

$$T_{sm} = 2\,F_y\,t_w\,y_o \qquad (31)$$

with its resultant at a distance $y_o/2$ from the mid-depth of the steel section. The longitudinal equilibrium of horizontal forces gives

$$C = T_{sm} = 2\,F_y\,t_w\,y_o \qquad (32)$$

The distance of the PNA from the mid-depth of the steel section is

$$y_o = \frac{C}{2\,F_y t_w} = \frac{C(d - 2t_f)}{2t_w(d - 2t_f)F_y} = \frac{C}{2\,P_{yw}}(d - 2\,t_y) \qquad (33)$$

where $(d - 2\,t_f)$ is the height of the web and P_{yw} is the web yield force.

The plastic moment of the composite beam is next obtained by taking moments about the mid-depth of the steel section as

$$\begin{aligned} M_n &= C\,e + M_p - T_{sm}\,\frac{y_o}{2} \\ &= C\left[\frac{d}{2} + Y_2\right] + M_p - C\,\frac{C}{4\,P_{yw}}(d - 2\,t_f) \\ &= C\,Y_2 + M_p + 0.5\,C\,d - \frac{C^2}{P_{yw}^2}\,M_{pw} \end{aligned} \qquad (34)$$

where $M_{pw} = \frac{1}{4}(d - 2t_f)^2\,t_w F_y$ is the web plastic moment.

The PNA will remain in the web for $0 \le y_o < (d - 2\,t_f)/2$. Equation 33 shows this condition is equivalent to $0 \le C < P_{yw}$.

Again, if $S < S^*$, the beam is a partially composite beam and the depth of the concrete compressive stress block a is less than the slab thickness t_c. On the other hand, if $S \ge S^*$, the beam is a fully composite beam and the entire slab thickness t_c is in compression. The results of Sects. 5 and 6 are summarized in Tables 1 and 2, and permit classification as a composite beam based on the relative strength of its elements.

DESIGN MOMENTS OF COMPOSITE BEAMS

In the Load and Resistance Factor Design Approach, design moments M_d are obtained from nominal moments M_n by multiplying them with a resistance factor ϕ_c, which is specified as 0.85 for composite beams. Thus,

$$M_d = \phi_c\,M_n \qquad (35)$$

resulting in the following relations (however, note the order of the three cases is reversed for the convenience of construction of design charts):

Case c: Plastic Neutral Axis in Web (Fig. 6)

From Eqs. 34 and 35, write

$$\begin{aligned} M_{dl} &= (\phi_c Y_2)\,C \\ &\quad + \left[(\phi_c M_p) + (0.5\phi_c d)\,C - \left\{ \frac{\phi_c M_{pw}}{P_{yw}^2} \right\} C^2 \right] \\ &= A_o C + [A_{11} + A_{12}C - A_{13}C^2] \end{aligned} \qquad (36)$$

where A_o is a constant for an assumed value of the parameter Y_2, while A_{11}, A_{12} and A_{13} are, for a given section and steel, also constants. Eq. 36 is valid for $0 \le C < P_{yw}$.

Case b: Plastic Neutral Axis in Flange (Fig. 5)

From Eqs. 30 and 35,

$$\begin{aligned} M_{d2} &= (\phi_c Y_2)\,C + \left[\left\{ 0.5\phi_c T^* \left(d - \frac{T^*}{2P_{yf}}\, t_f \right) \right\} \right. \\ &\quad \left. + \left\{ \phi_c\,\frac{T^*}{2\,P_{yf}}\, t_f \right\} C - \left\{ \frac{\phi_c t_f}{4\,P_{yf}} \right\} C^2 \right] \\ &= A_o C + [A_{21} + A_{22}C - A_{23}C^2] \end{aligned} \qquad (37)$$

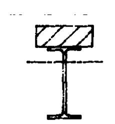
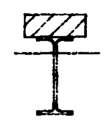
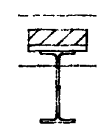
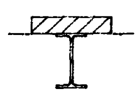
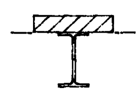
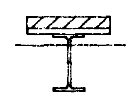
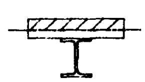
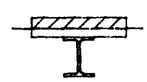
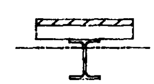
where A_{21}, A_{22} and A_{23} are another set of constants for a selected steel section and A_o is as defined in Eq. 36.

Table 1. Classification of Composite Beam Based on Relative Strength of Its Elements

		$S > S^*$	$S = S^*$	$S < S^*$
I.	$C^* < T^*$	1. Fully composite 2. Inadequate slab 3. PNA in steel 4. Use charts PFS, Ref. 11 or LRFD	1. Fully composite 2. Inadequate slab 3. PNA in steel 4. Use charts PFS, Ref. 11 or LRFD	1. Partially composite 2. Inadequate slab 3. PNA in steel 4. Use charts PFS or LRFD
II.	$C^* = T^*$	1. Fully composite 2. Adequate slab 3. PNA at TFL 4. Use charts PFS, FC, Ref. 11 or LRFD	1. Balanced design 2. Adequate slab 3. PNA at TFL 4. Use charts PFS, FC, Ref. 11 or LRFD	1. Partially composite 2. Adequate slab 3. PNA in steel 4. Use charts PFS or LRFD
III.	$C^* > T^*$	1. Fully composite 2. Adequate slab 3. PNA in slab 4. Use charts FC or Ref. 11	1. Fully composite 2. Adequate slab 3. PNA in slab 4. Use charts FC or Ref. 11	1. Partially composite 2. Adequate slab 3. PNA in steel 4. Use charts PFS or LRFD

C^* = Maximum compressive force of concrete slab = $0.85\ f'_c\ b\ t_c$
T^* = Yield force of steel section = $A_s\ F_y$; $S^* = \min(C^*, T^*)$
S = Strength of connectors between zero and maximum moment points = $N\ Q_{nr}$
PNA = Plastic neutral axis
TFL = Top fiber of steel flange

Table 2. Classification of Composite Beam Based on Relative Strength of Its Elements

		S>S*	S=S*	S<S*
I	C* < T*	 C* < T* < S C* < S < T*	 S = C* < T	 S < C* < T
II	C* = T*	 C* = T* < S	 S = C* = T*	 S < C* = T*
III	C* > T*	 T* < C*< S T* < S < C*	 S = T* < C*	 S < T* < C*

Eq. 36 is valid for $P_{yw} \leq C < T^*$

To summarize, the design moment for the first two cases (Eqs. 36 and 37) could be put in the single format,[17]

$$M_d = M_{dY} + M_{do} \tag{38}$$

where M_{do} is that part of the design moment corresponding to the hypothetical case where $Y_2 = 0$†. For a given steel section, M_{do} is a function of the concrete force C only. M_{dY} represents the influence of the distance Y_2 on the design moment M_d. It is independent of any terms related to the steel section and is a linear function of the concrete force C for an assumed value of Y_2.

Case a: Plastic Neutral Axis in the Concrete Slab (Fig. 4)

From Eqs. 19 and 35,

$$M_{d3} = [\phi_c 0.5d\ T^*] + [\phi_c\ T^*]Y_2 \tag{39a}$$

or

$$M_d = B_o + B_1\ Y_2 \tag{39b}$$

where B_o and B_1 are yet another set of constants for a selected steel section. The above equation is valid for C^* ³ T^* and $S > S^*$.

Equation 39b indicates that for a selected steel section, the design moment is a linear function of the parameter Y_2.

DESIGN CHARTS

The design charts are divided into two groups. Charts PFS could be used for the design of *Partially* composite beams and *Fully* composite beams with the PNA in the *Steel* section. The charts FC could be used for the design of *Fully* composite beams with the PNA in the *Concrete* slab. (Tables 1 and 2).

It should be noted the eight charts PFS convey essentially all the information given by the composite beam design strength tables included in the LRFD Manual.

CHARTS PFS

As seen from Eqs. 36, 37 and 38 for design moment M_d, that if a specified steel yield stress F_y is selected first, and concrete force C is considered as a parameter, then it is possible to plot curves of M_{do} against C for a wide range of rolled-steel, wide-flange, beam-sections. Also, the relationship between C and M_{dY} can be shown on the same plot by drawing a series of straight lines through the origin corresponding to a set of pre-selected values for Y_2. In this paper, values of 2, 3, 4, 5, 6 and 7 in. for Y_2 are considered. The entire range of curves can then be replotted for other sets of rolled shapes and other steels. The resulting set of curves are given as design charts PFS-1 to PFS-8, for F_y = 36 and 50 ksi. Where the curves are in full lines the plastic neutral axis is in the flange of the steel section. The broken curve to the left of a continuous line indicates that the PNA is in the web of the steel section.

On any curve, the point represented by a solid square corresponds to the abscissa $C = 0.25\ A_s\ F_y = S_{min}$, which is a practical minimum value for the shear connector capacity suggested by the LRFD Manual. To the left of this point, it is suggested the beam be designed as a non-composite beam with a resistance factor of 0.9 (instead of the value of 0.85 used in the construction of the curves).

It is worthwhile to note the correspondence between several points on these curves and the designations for the position of the PNA, used in the LRFD Manual composite design strength tables. Thus, a solid circle on a curve corresponds to the case where the PNA is at the top flange of the steel section (position designated as TFL in the LRFD tables). An open circle on a curve corresponds to the case where the PNA is at the bottom flange of the steel section (position designated as BFL in the LRFD tables). The solid square corresponds to the position designated as Position 7 in the LRFD tables.

The broken horizontal line to the right of the solid circle indicates that the PNA is above the steel section and hence this part of the curve in PFS charts is no longer valid for that section and the values of C (try charts FC described next).

CHARTS FC

As seen from Eqs. 39a and 39b, that for a specified section and steel yield stress F_y, the design moment M_d varies linearly with the parameter Y_2. All these lines are shown in Charts FC-1 to FC-4 for all rolled-steel beam sections for F_y = 36 ksi and 50 ksi. As mentioned earlier, these charts are valid for the case where the PNA is in the slab, i.e., for fully composite beams with adequate slab.

The use of these charts for the design of composite beams will be described in the next two sections.

DESIGN PROCEDURE

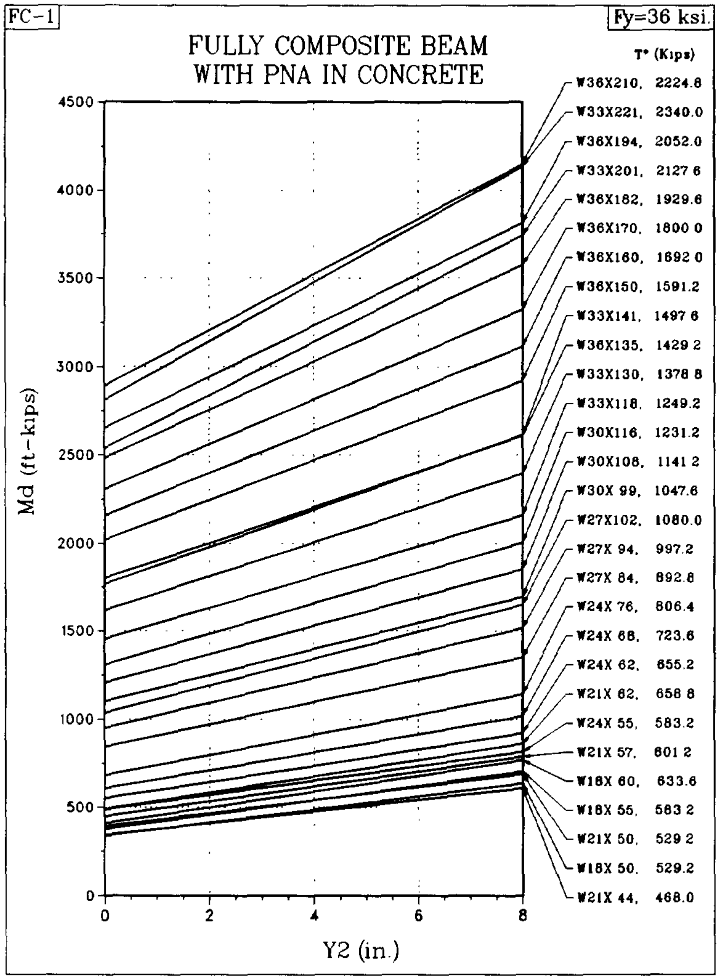
The design of simply supported, fully or partially composite beams using rolled-steel sections and ribbed-metal deck perpendicular to the beam, is summarized:

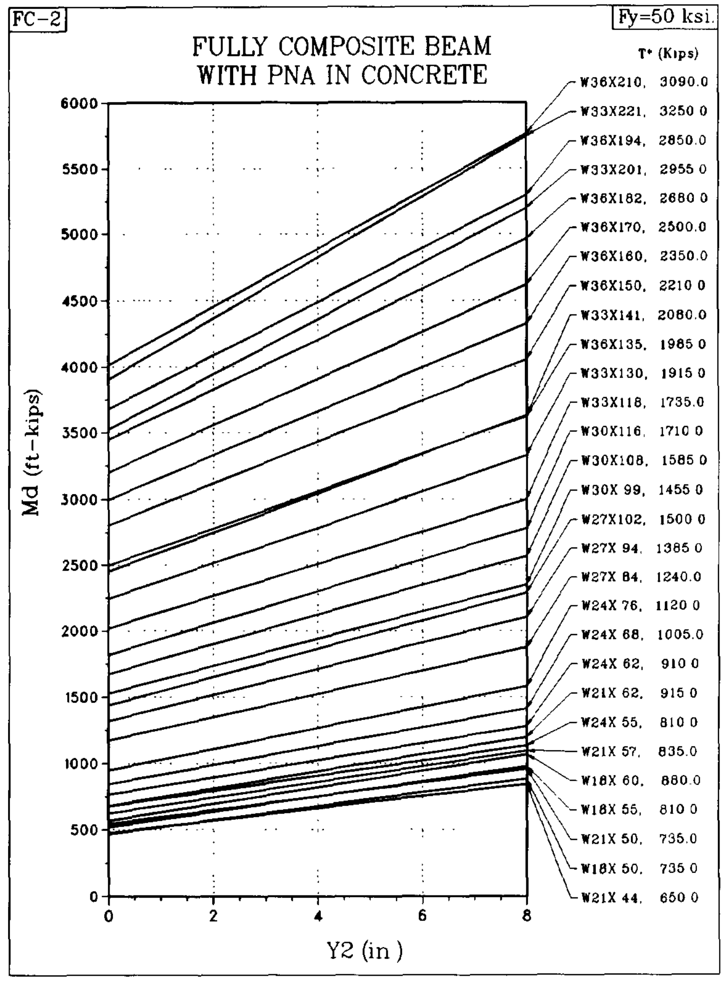
1. Rib dimensions h_r, w_r and slab thickness t_c

The design of the composite floor deck is performed following the manufacturer's recommendations and test data on proprietary decks. The metal deck and shear studs should satisfy these general rules:

a. The decking rib height shall not exceed 3 in., that is, $h_r \leq 3.0$ in.
b. The rib width shall not be less than 2 in., that is, $w_r \geq$ 2.0 in.
c. Slab thickness above the steel deck shall be not less than 2½ in., that is $t_c \geq$ 2½.

†*That is, the slab compressive force C is concentrated at the top flange of the steel section.*





d. Stud shear connectors shall be ¾ in. or less in diameter, that is $d_{sc} \le$ ¾ in.

e. Stud shear connectors shall extend at least 1½ in. over the top of steel deck, that is $H_s \ge h_r$ + 1½ in.

Also, a general good practice is to limit the span of the composite deck floor to 32 times the total depth (= $h_r + t_c$) of the floor section.

At this stage, we know the parameter Y_2 lies between $h_r + \dfrac{t_c}{2}$ and $h_r + t_c$.

2. Effective width of slab *b*

According to the LRFD Specification, Sect. I3-1, the part of the effective width of the slab on each side of the centerline of the steel beam shall not exceed:

a. One-eighth of the beam span, center to center of supports;

b. One-half the distance to the center line of the adjacent beam; or

c. The distance from the beam center line to the edge of the beam.

At this stage, the three basic concrete parameters (f_c', t_c and b) are all known. Hence, the parameter $C^* = 0.85\ f_c'\ b\ t_c$ can be calculated.

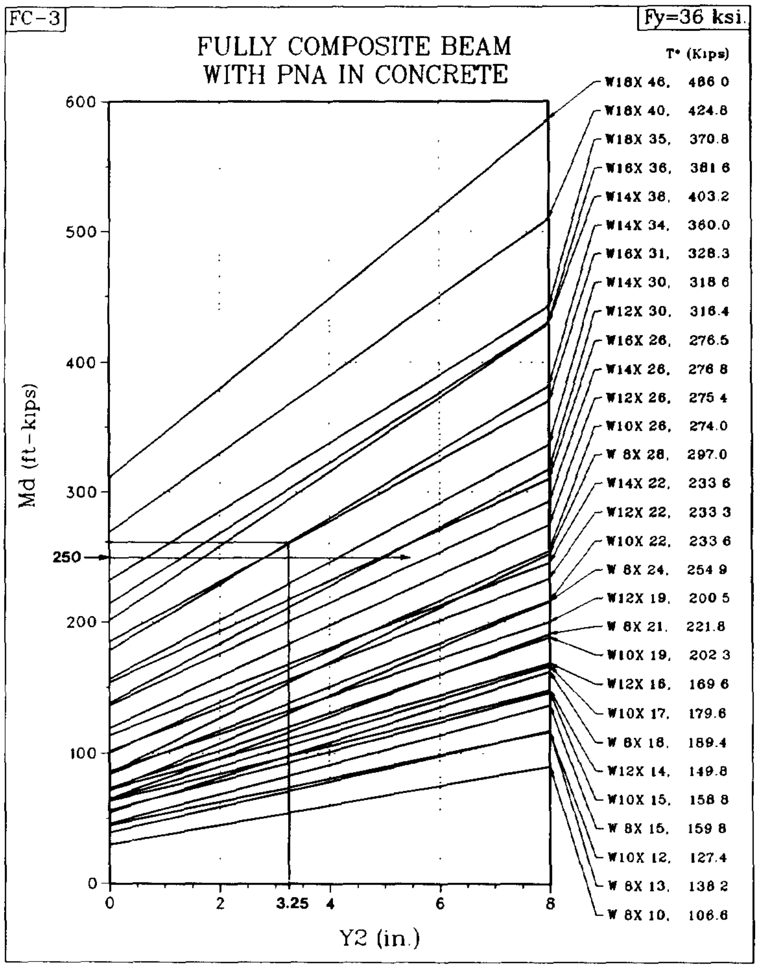
3. Bending moment M_r

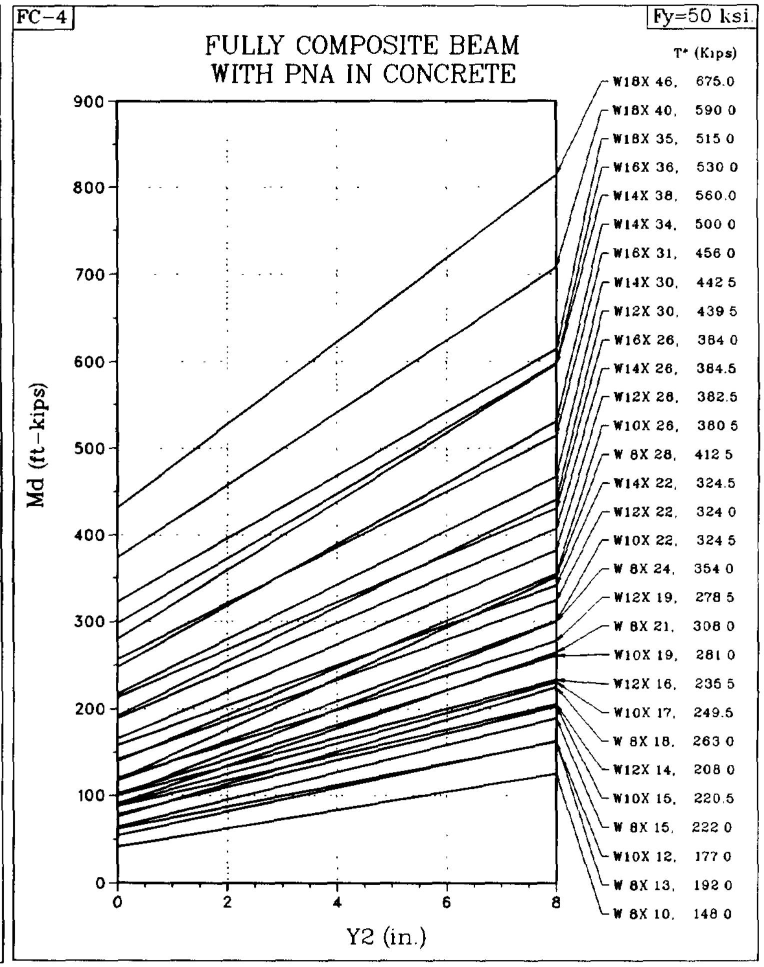
It is well known[3-9] that the effect of the method of construction (whether shored or unshored) on the plastic moment of a composite beam is negligible. The required flexural strength can therefore be calculated on the assumption that the composite section resists all the loads (self weight of the steel beam, concrete slab, all permanent loads and live loads). For example, if w_D and w_L are the dead load and live load per unit length of the beam, respectively, and L is the span, then

$$M_r = (1.2w_D + 1.6w_L)\frac{L^2}{8} \qquad (40)$$

4. Beam size selection

The beam size is selected, using the design charts, by trial and error. By referring to the design charts presented in this paper, select the one for the appropriate stress F_y. (The





procedure is explained here with the help of Fig. 7, an excerpt from the design chart PFS-4.) Plot the point Ⓐ corresponding to the value $M_d = M_r$ on the Y axis. Also locate the point Ⓑ corresponding to the value C^* on the abscissa.

Next, estimate a value a^o for the depth of the compressive stress block a (between 0 and $t_c/2$) and, hence, an estimate Y_2^0 for the parameter $Y_2 = h_r + t_c - a/2$. In the bottom part of the chart (Fig. 7) locate the line ① corresponding to the particular value of $Y_2 = Y_2^o$, by interpolation if necessary. Through the point Ⓐ draw a straight line ② parallel to the straight line ①. Let P_1, P_2, ... P_i, ... be the points of intersection of line ② with different curves shown in Fig. 7. We observe that the vertical intercept between any one of these points and the diagonal line ① is equal to $M_{dY} + M_{do} = M_d = M_r$. So, all the points P_i are possible candidates for the design, subjected to the following comments and restrictions:

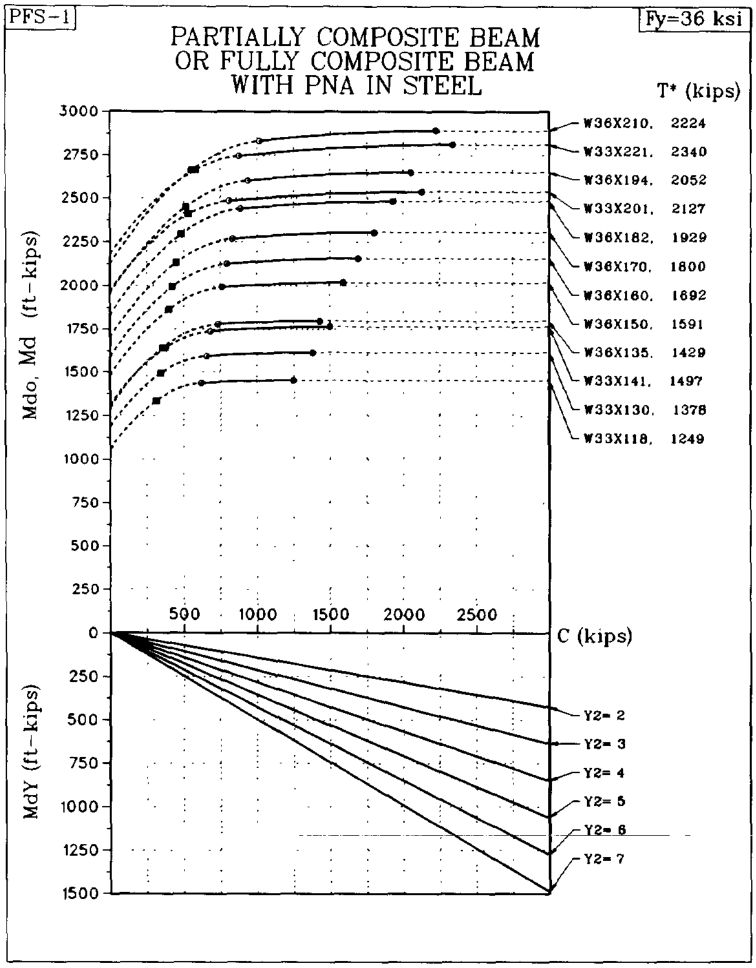
a. If the line ② intersects a curve to the left of the solid square on that curve, the shear connector capacity $C = S$ using the corresponding section is less than the minimum value S_{min} recommended by the LRFD Manual and preferably should not be considered for design as a composite beam.

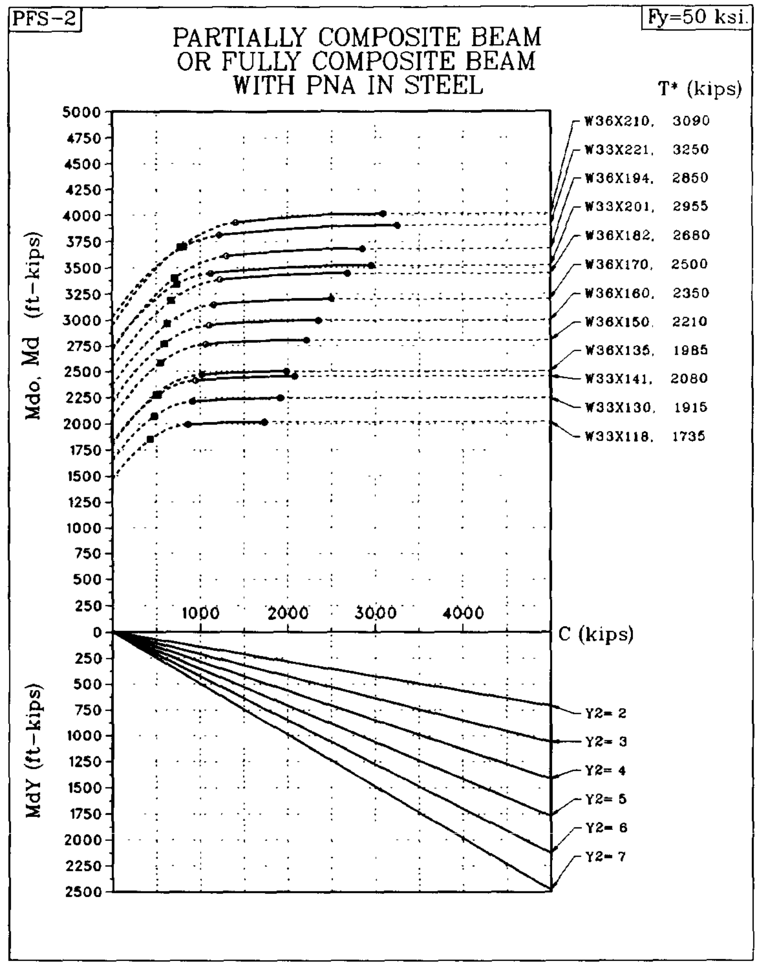
b. If a point of intersection (P_i) is to the right of the point Ⓑ, the corresponding section and sections below it are not candidates for design as they violate the condition $c = \min(C^*, T^*, S)$.

c. If a point of intersection (P_i) coincides with the solid circle on a curve and if, in addition, the abscissae of (P_i) and Ⓑ are same, then the corresponding design results in a balanced design as we have $C = C^* = T^* = S^* = S$.

d. If the point of intersection (P_i) is at or to the left of the solid circle on a curve, then that section results in a partially composite design. The corresponding beam is heavier than the one corresponding to the balanced design, while the shear connectors required are fewer.

e. If a point of intersection (P_i) is at or to the right of the solid circle on a curve the corresponding design possibly results in a fully composite beam with the PNA





in the slab. As mentioned earlier, the charts PFS are no more valid to the right of the solid circle, however. The charts FC should be used if the designer intends to use fully composite design. Such a design results in lighter steel beam but with more shear connectors, compared to the balanced design.

f. Whether a balanced, fully composite or partially composite design results in the most economical beam depends on the relative cost of steel, shear connectors and labor for welding connectors. It is generally felt, however, that in the USA partially composite designs generally result in the most economical designs.

For the selected design corresponding to point ⓙ calculate the actual values a^l of the depth of the compression stress block from

$$a^l = \frac{C}{0.85\, f'_c b} \qquad (41)$$

If a^l is not approximately equal to the initially assumed value a^o, the calculations described in this section are repeated with a new value of a. Also, before the design can be finalized, shear connectors are to be designed and checks are to be made for deflection, shear and construction stage strength.

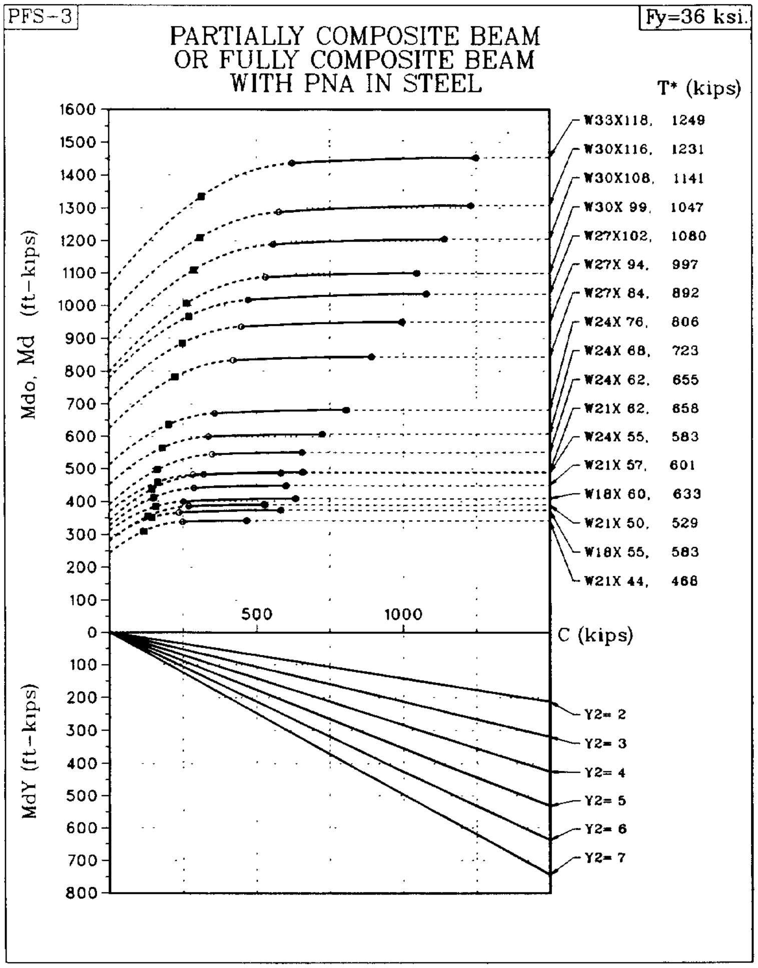
5. Shear connector design

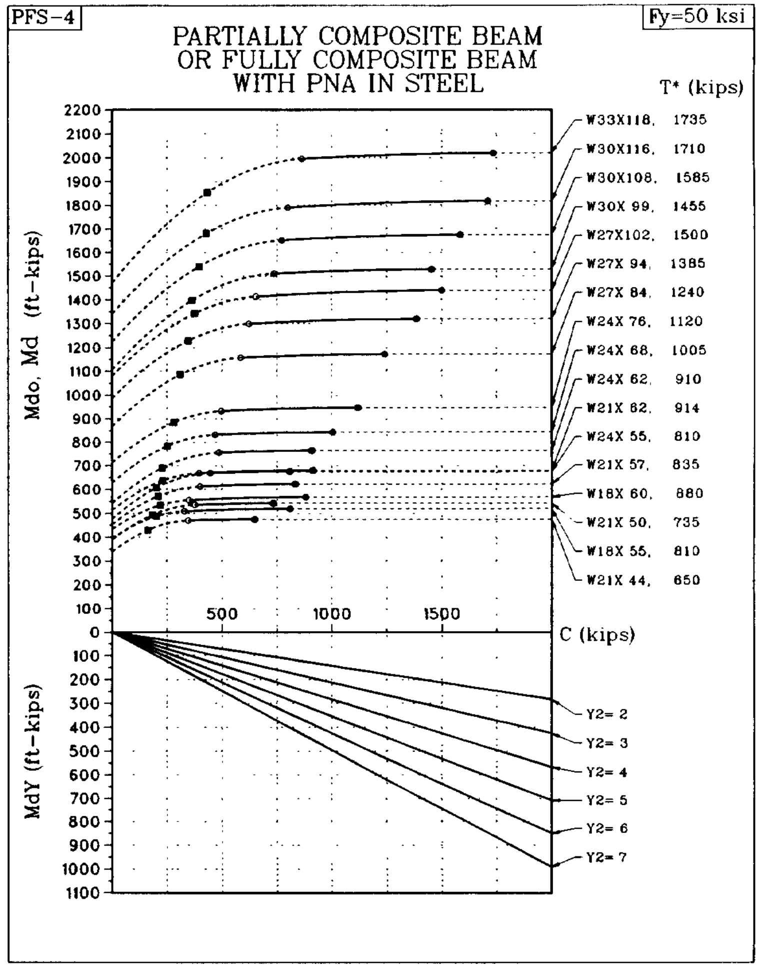
The ultimate horizontal shear force to be transferred from the deck to the steel beam, or vice versa, is the value C corresponding to the beam selected in the previous section.

If Q_{nr} is the nominal strength of shear connector corrected for the reduction factor for deck ribs (see Eq. 3), then the number of connectors between the point of maximum positive moment and the point of zero moment to either side is given by

$$N = \frac{C}{Q_{nr}} \qquad (42)$$

These shear connectors may be spaced uniformly over that length (i.e. L/2), with the proviso that spacing is compatible with the pitch of the ribs when metal deck is used.





6. Shear design

No significant increase of the shear capacity is achieved by composite action. So the shear force check is performed as for non-composite beams, i.e., the web of the steel beam resists the full shear force. The effect of coping, if provided, should be considered in these calculations.

7. Deflections

When a composite beam has been designed by ultimate strength methods described in this paper, checks must be made to assure that its deflection is not excessive at service loads. This can be done by using the tables of lower bound moments of inertia of composite sections given in the LRFD Manual. If deflections so calculated are acceptable, a complete elastic analysis, using the transformed section method, could be avoided.

8. Strength during construction

When shoring is not used during construction, the steel beam alone must resist all loads applied before the concrete has hardened enough to provide composite action. LRFD Specification Sect. I3.4 requires that 75% of the compressive strength f_c' of the concrete must be developed before the composite action may be assumed.

Load factor for wet concrete is preferably taken as 1.6 (as for live loads). Construction loads and their load factors should be determined by the designer for individual projects. The flexural strength of the steel section is determined in accordance with the requirements of LRFD Sect. F1.

DESIGN EXAMPLES

For comparative purposes, the following example taken from the LRFD Manual is reworked using the design charts PFS.

Example 1:

Select a beam, with F_y = 50 ksi, required to support a service dead load of 90 psf and a service live load of 250 psf. The beam span is 40 ft and the beam spacing is 10 ft. Assume a 3-in. metal deck with a 4.5-in. thick concrete slab of 4-ksi normal weight concrete (145 pcf). The stud reduction factor

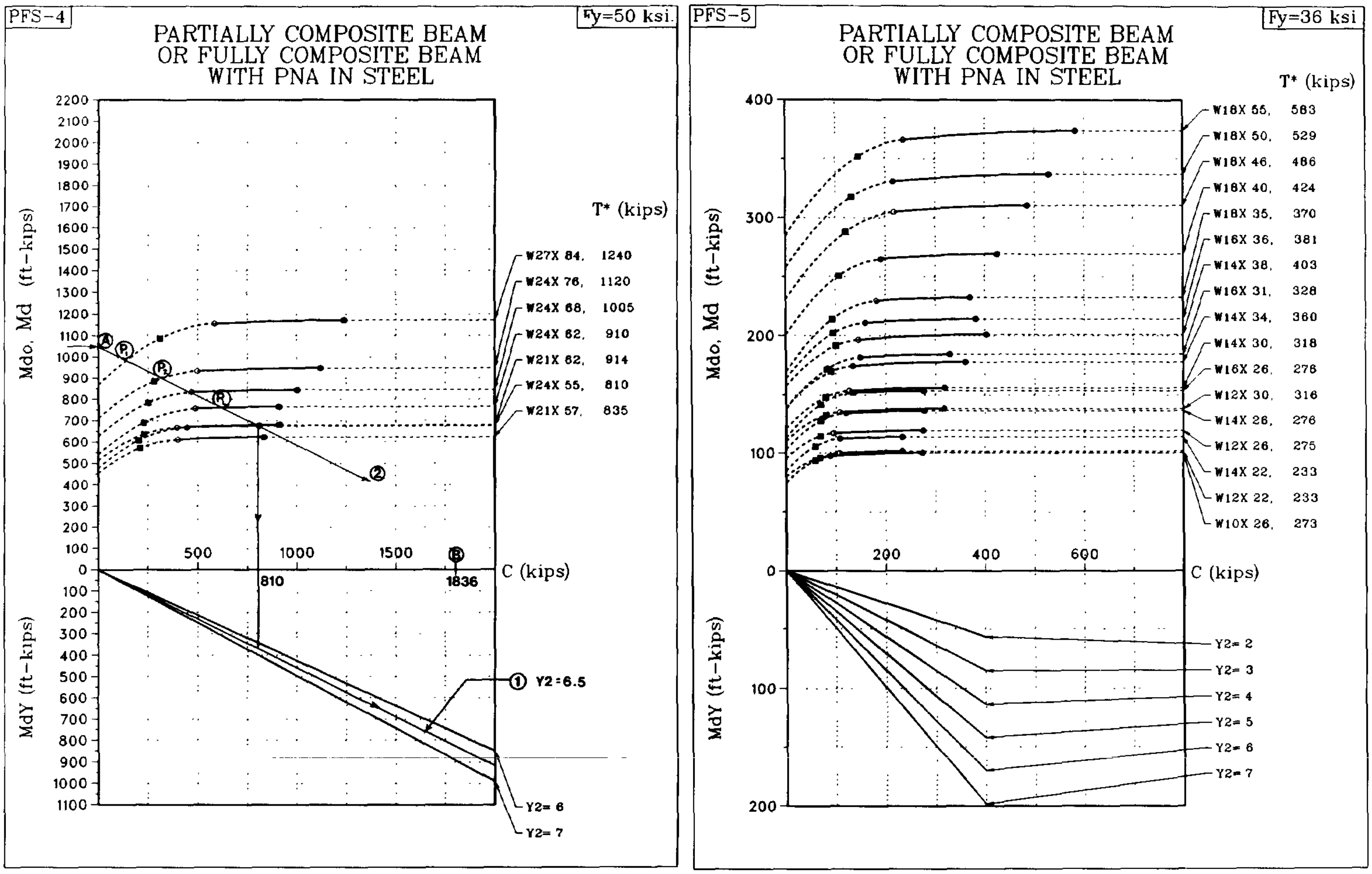


Fig. 7. Example 1

is 1.0. Unshored construction is specified. Determine the beam size.

Solution:

a. Data:

h_r = 3 in. t_c = 4.5 in.

Y_c = 7.5 in.

L = 40 ft s = 10 ft

$$b = \min \frac{40}{4} \times 12, 10 \times 12 = 120 \text{in.}$$

f_c' = 4.0 ksi

$C^* = 0.85\ f_c' b t_c = 0.85\ (4.0)\ 120\ (4.5)$

$= 408\ (4.5) = 1{,}836$ kips

It is seen from the last relation that the compressive force for each inch thickness of compressive stress block is 408 kips. That is

$$C_{(a=1")} = 408 \text{ kips}$$

b. Determine moments:

	Service Load (K/ft)	($L\ F$)	Factored Load (K/ft)
DL 10(0.09) =	0.9	1.2	1.08
LL 10(0.25) =	2.5	1.6	4.00
	3.4		5.08

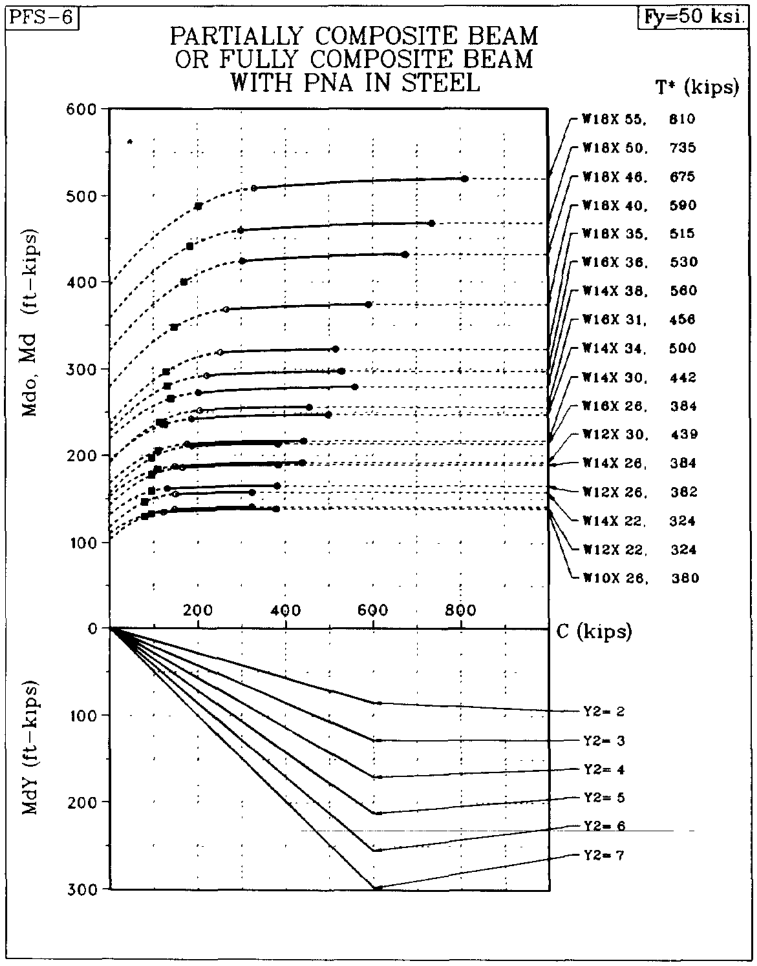
Required moment = 5.08 $(40)^2/8$ = 1,016 kip-ft
Let us use a rounded value of 1,050 kip-ft

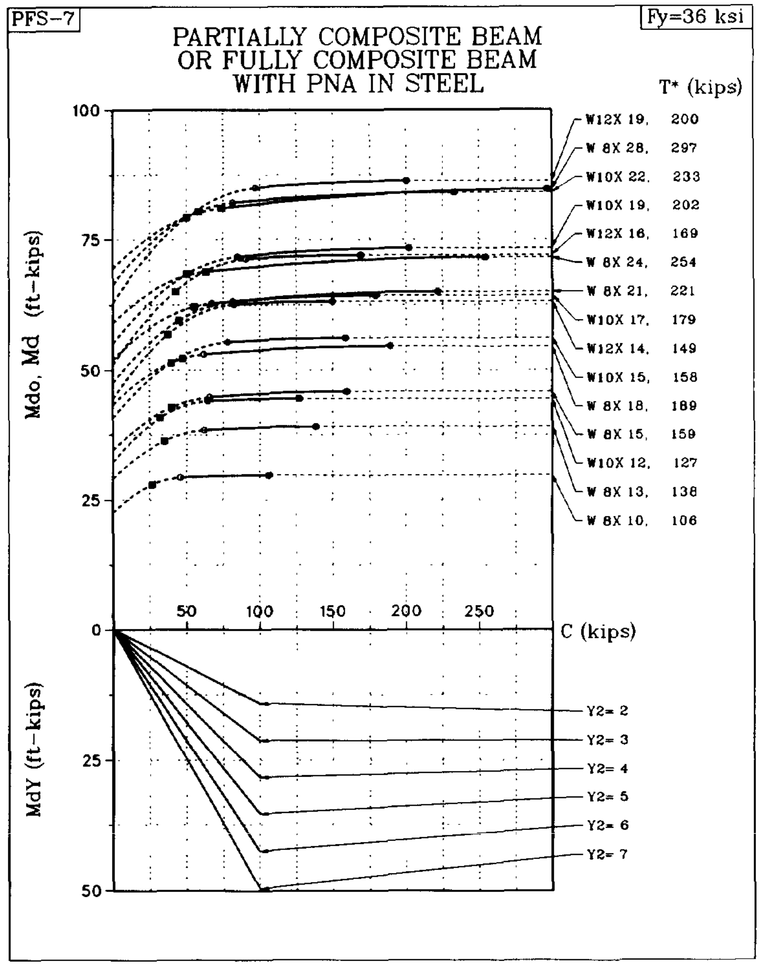
c. Beam Size:

Select chart PFS-4 corresponding to F_y = 50 ksi and plot the point Ⓐ corresponding to M_d = 1050 kip-ft, on the M_d axis (Fig. 7). The point Ⓑ corresponding to $C = C^*$ = 1836 kips is marked on the C axis next. A preliminary estimate for a can be taken as a^o = 2 in. resulting in a value for Y_2 of

$$Y_2^o = Y_c - a^o/2 = 6.5 \text{ in.}$$

The line ① corresponding to Y_2^0 = 6.5 is next drawn and then the line ② parallel to ① passing through the point Ⓐ. All sect-





ions corresponding to the curves intersected by the line ②, provided the actual Y_2 value equals the assumed value of Y_2^0 = 6.5, deliver the required moment of 1050 ft kips. The abscissae corresponding to these points of intersection, give the value C for the compressive force in the concrete.

Note that as line ② intersects the curve for W27×84 to the left of the solid square, that section is not included in Table 3. Also, line ② intersects the curves for W21×57 and lighter sections to the right of the solid circle, so they are not included either in the above table. However, if the designer intends to use one of these lighter sections, he should use charts FC for fully composite beams (see next example).

For the last two sections of Table 3, a^l = 1.99 ≈ a^o indicating the value of Y_2^o = 6.5 in. used in constructing line ① is correct. So they satisfy the strength design requirements. Of these two, select the deeper and lighter W24×55 section with C = 810 kips.

If the designer intends to use even fewer shear connectors (smaller C), the first three sections could be reconsidered with revised values for a and Y_2 and repeat the procedure followed earlier.

Table 3

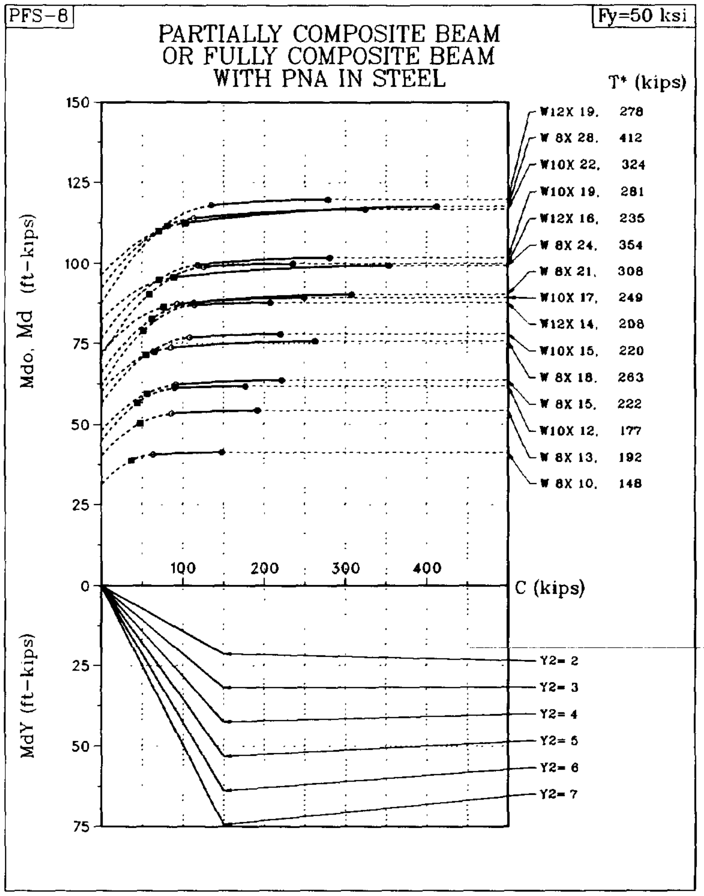
	C	$a^l = \frac{C}{0.85 f_c' b}$	T^*
Section	(kips)	(in.)	(kips)
W24 × 76	317	0.78	1,120
W24 × 68	472	1.16	1,005
W24 × 62	627	1.54	910
W21 × 62	810	1.99	914
W24 × 55	810	1.99	810

d. Design shear connectors:

Horizontal shear force to be transferred from the concrete slab to the metal beam is

$$S = H = C = 810 \text{ kips}$$

Select ¾-in. dia. headed studs.



Capacity of a stud in normal weight concrete, with $f_c' = 4$ ksi, is obtained from Table 12 of LRFD as 26.1 kips.

The stud reduction factor R is given as 1.0. So,

$$Q_{nr} = 26.1 \text{ kips}$$

and

$$N = \frac{C}{Q_{nr}} = \frac{810}{26.1} = 31.03$$

A W24×55, F_y = 50 ksi with 64 – ¾-in. dia. headed studs satisfies requirements for flexural design strength.

e. Check for strength at construction phase:

Construction loads:

		Service Load (K/ft)	L.F.	Factored Load (K/ft)
LL	10 (0.02)	0.20	1.6	0.32
Concrete	10 (0.075)	0.75	1.6	1.20
DL	10 (0.09-0.075)	0.15	1.2	0.18
		1.10		1.70

Note an assumed construction live load of 20 psf and that the load factor for wet concrete is taken as 1.6. Required moment = 1.70 $(40)^2/8$ = 340 ft kips.

Assume the metal deck connected to the steel beam by stud connectors and spot welds provides adequate lateral support for the steel beam and design it per LFRD Sect. F1. From Tables on p. 3-41 of the LRFD Manual.

$$\phi_b M_p = 503 \text{ ft} > 340 \text{ ft kips}$$

So the design is adequate.

As a second example, consider that from Ref. 11 and rework it with the help of charts FC.

Example 2:

Design the lightest weight composite section, with a flat soffit slab, for use as an interior floor beam of an office building, using the following data:

Simple span, L = 28 ft	Live load = 125 psf
Beam spacing, s = 8 ft	Partitions = 25 psf
Slab thickness, t = 4 in.	Ceiling = 8 psf

Normal weight concrete, f_c' = 3 ksi

A36 steel, F_y = 36 ksi

Solution:

a. Determine moments:

4 in. slab	(4/12) (8) 0.15	= 0.40 kip/ft
Ceiling	(0.008) 8	= 0.06 kip/ft
Steel beams (assumed)		= 0.04 kip/ft
Total dead load, w_D		= 0.50 kip/ft
Live load	(0.125) 8	= 1.00 kip/ft
Partitions	(0.025) 8	= 0.20 kip/ft
Total live load, w_L		= 1.20 kip/ft

Required moment $M_r = (1.2\, w_D + 1.6\, w_L)L^2/8$
$= 2.52\,(28)^2/8$
= 250 ft kips

b. Determine C^*:

Effective width of slab, for interior beam
b = min [L/4, s] = min [0.25 (28)12, 8(12)]
= min [84, 96] = 84 in.

Capacity of concrete slab in compression
= $C^* = 0.85\, f_c'\, b\, t$ = 0.85 (3) 84 (4)
= 857 kips

Capacity of concrete for each inch of compression block thickness

$$C_{(a=1)} = 0.85\,(3)\,84\,(1) = 214 \text{ kip/in.}$$

c. Beam size:

Select chart FC-3 corresponding to F_y = 36 ksi and draw a horizontal line corresponding to $M_d = M_r$ = 250 ft kips.

For a 4-in. thick flat soffit slab, Y_2 lies between 2 in. (when a = 4 in.) and 4 in. (when $a \approx 0$).

For sections that cross the 250 ft kip line in this region the value of T^* is seen to be about 320 kips. As the capacity of concrete for each inch of compression block thickness is 214 kips, a^o is equal to 320/214 = 1.5 in., and Y_2^0 = 4 − 1.5/2 = 3.25 in.

As can be seen from the construction on FC-3, it is seen that W14×34 with T^* = 360 kips and W16×31 with T^* = 328 provide the required strength. Select the deeper, lighter W16×31 section.

d. Design shear connectors:

Horizontal shear force to be transferred from the concrete slab to the steel beams is

$$S^* = \min [C^*, T^*] = \min [857,328] = 328 \text{ kips}$$

Use ¾-in. dia. × 3-in. headed stud connectors. From Table 12 of the LRFD, the horizontal shear load for one connector in 3 ksi normal weight concrete is 21.0 ksi. So the number of connectors between the point of maximum moment (i.e., mid span) and the point of zero moment to either side (i.e., support point) is:

$$N = \frac{328}{21.0} = 15.6 \text{ say } 16$$

which is 32 connectors for the entire span.

CONCLUSION

The paper presents charts that greatly simplify the design of composite beams using the new LRFD Specification. They are equally applicable to composite beams with flat soffit slab, with haunched slab or with composite metal decking wherein the ribs run perpendicular to the beam. Both partially composite and fully composite beams are included in the study.

Unlike the design tables, the design charts provided in this paper are powerful, because in using them it is possible to isolate a range of alternate designs that satisfy the LRFD strength design criteria. The most desirable, or the most economical design could then be selected from this set using engineering judgement.

The charts should be of interest as a teaching aid, and familiarity with them should contribute to overall feel of the composite beam design problem to students and young engineers.

Acknowledgements

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NOMENCLATURE

A_s = Area of the steel section, in.^{2}
A_{cs} = Area of the steel section in compression, in.^{2}
a = Depth of the compressive stress block in concrete, in.
b = Effective width of slab, in.
b_f = Width of steel flange, in.
C = Compressive stress resultant in slab, kips = min[C^*, T^*, S].
C^* = Maximum compressive force of the concrete slab (= 0.85 $f_c' b t_c$).
C_s = Compressive stress resultant in steel section, kips.
d = Depth of steel section, in.
e, e_o, e_m = Lever arms, in.
F_c = Equivalent yield stress of concrete (= 0.85 f_c'), ksi.
F_u = Tensile strength of connector steel, ksi.
F_y = Yield stress of steel, ksi.
f_c' = Specified compressive strength of concrete, ksi.
H = Horizontal shear force between the slab and beam, kips.
H_s = Length of stud shear connector, in.
h_c = Height of web clear of fillets, in.
h_h = Height of haunch, in.
h_r = Rib height, in.
L = Span of beam.
M_d = Design moment of composite beam (= $\phi_c M_n$).
M_n = Nominal resisting moment of the composite beam.
M_p = Plastic moment of steel section, kip-in.
M_r = Required moment capacity under factored loads.
M_{do} = Design moment of a composite section, with Y_2 set equal to zero.
M_{dY} = Influence of Y_2 on the design moment, M_d.
M_{pw} = Web plastic moment, kip-in.
N = Number of shear connectors between zero and maximum moment.
N_r = Number of stud connectors in one rib at a beam intersection ($\leq$ 3 in calculations).
N^* = Number of connectors N for a fully composite design.
P_{yf} = Flange yield force, kips.
P_{yw} = Web yield force, kips.
Q_n = Nominal strength of shear connector, kips.
Q_{nr} = Strength of a shear connector in a deck rib, kips.
R = Strength reduction factor for connectors in a rib.
S = Strength of connectors between zero and maximum moment points, kips (=$N Q_{nr}$).
S_{min} = Suggested minimum strength of connectors, kips.
S^* = Connector strength S for fully composite design, kips.
s = Spacing of beams, in.
T = Tensile force resultant in composite section, kips.
T^* = Maximum tensile yield strength of steel section, kips. (= $A_s F_y$)
t = Thickness of a flat soffit slab, in.
t_c = Thickness of concrete slab above rib, in.
w = Unit weight of concrete, pcf
w_r = Average width of deck rib, in.
Y_c = Distance from the top of steel beam to top of concrete, in.
Y_2 = Distance from the top of steel beam to the compression force C in concrete, in.
y = Depth of steel flange in compression (Case b), in.
y_o = Distance of plastic neutral axis from mid-depth of steel section (Case c), in.
ϕ_b = Resistance factor for non-composite beams (= 0.90)
ϕ_c = Resistance factor for composite beams (= 0.85).