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Exploring Control Design Variables

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ABSTRACT

This paper suggests a method of control system parameter selection based on a set of parametric plots of closed-loop system performance. Here, the performance measures of rise time, settling time, percent overshoot, root sensitivity, gain margin, and phase margin are investigated as functions of the forward loop gain. The explicit functional dependence of these performance indices is summarized in plots that highlight the control system design space as well as the tradeoffs between feasible system parameters. An example problem demonstrates how the information serves as an aid in satisfying design specifications and uncovering design tradeoffs.

1. INTRODUCTION

Control system design invariably involves tradeoffs, *e.g.*, an increase in loop gain to reduce the steady-state error is generally accompanied by deterioration of the transient performance (in terms of increased overshoot in the response and reduced damping of the dominant poles). Closed-loop specifications are most often given in terms of steady-state error and step response parameters such as rise time, peak time, settling time, peak overshoot, *etc.* Relative stability measures of gain margin and phase margin, typically associated with frequency-domain specifications, may also be given.

This paper promotes the use of computer-based tools to generate parametric plots to aid in the process of control system tuning and design. The premise of the method is that various performance measures may be plotted as functions of control design parameters to highlight the control system design space as well as identify tradeoffs among feasible system parameters. Many performance metrics may be considered when designing control systems (*e.g.*, [1]). In this paper, a theme problem is used to demonstrate the graphically-based parametric design approach. We simultaneously explore rise time, settling time, percent overshoot, root sensitivity, gain margin and phase margin as functions of the forward loop gain.

1.1. Overview of Gain Plots

Geometric perspectives by Nyquist [2], Bode [3], and Evans [4] have lead to many important developments in classical control theory of linear time-invariant (LTI) systems. These graphically-based methods remain key control analysis and design tools even in the post-computer age. The advent of the computer has enabled the rapid simulation of closed-loop responses as well as the near-automatic generation of Nyquist diagrams, Bode plots, and Evans' root locus plots. It has also extended the classical methods considerably. For example, multivariable root locus plots can be generated as readily as single-input, single-output (SISO) root locus plots.

The power and availability of software tools has prompted the consideration of a new geometric perspective of the Evans root locus [5]. The resulting plots, called gain plots, show the explicit variation of eigenvalue magnitudes and angles with respect to a scalar system parameter, such as forward gain [6]. Eigenvalue magnitudes *vs.* gain are portrayed in a magnitude gain plot using a log–log scale and eigenvalue angles *vs.* gain are displayed in an angle gain plot using a semi–log scale (with the logarithms being base 10). Although these plots employ eigenvalue magnitude and angle as the functions of interest, similar plots depicting classical control performance measures may be generated. As seen in Section 2, these plots are rich in design implications.

1.2. Example

The example chosen for this paper is adapted from Problem DP6.3 of the textbook by Dorf [7]. The example considers the design of a Mars vehicle robot controller, shown in Figure 1. The task is to select the forward loop gain, k, cascaded with the PID controller shown in the block diagram of Figure 1 that meets the following specifications:

1. Maximum Percent Overshoot to Unit Step = 15% (This ensures that the robot arms do not substantially overshoot their target destination.)
2. 2% Settling Time to Unit Step < 3 seconds. (This guarantees a reasonably rapid target lock.)
3. Rise Time to Unit Step > 0.25 seconds. (This limits the maximum power requirement of the actuators.)
4. Phase Margin > 45°. (This is a frequency domain specification providing the design with robustness.)
5. Gain Margin > 8 dB. (This is a frequency domain and gain domain specification providing the design with robustness.)
6. Maximum Root Sensitivity (for both real and imaginary components) < 2. (This is a robustness specification that ensures limited closed-loop dynamic changes to gain and parameter variations and uncertainties.)

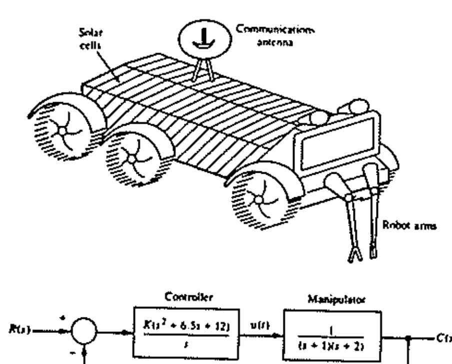

Figure 1. Mars vehicle robot and block diagram.

2. GRAPHICAL ANALYSIS

This section presents a variety of graphical representations portraying closed-loop system specifications as a function of forward loop gain. Initially, each of the specifications is considered separately, and conclusions are drawn. Concurrent consideration of the graphs, the topic of the following section, permits the determination of the feasibility of the specifications as well as the identification of tradeoffs.

2.1. Root Locus

Figure 2 presents the root locus for the robot control system of Figure 1. There are open-loop poles at $s = 0, -1$, and -2 and a pair of complex conjugate transmission zeros at $s = -3.25 \pm 1.2j$. There are also two breakout points and one break-in point. The root locus shows that for all positive values of gain the system is nominally stable. At high gains, two of the poles migrate to the finite zeros, essentially negating the dynamics of the two closed-loop poles and zeros. Also at high gain the "excess" pole traverses the negative real axis toward $-\infty$. The resulting high gain closed-loop system may be considered to behave as a first order system.

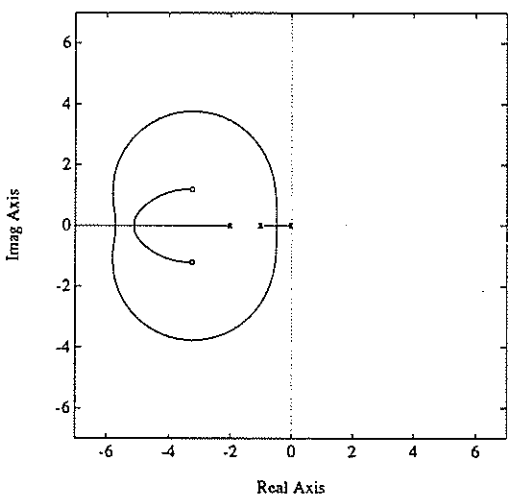

Figure 2. Root locus plot for the robot control system of Figure 1.

2.2. Gain Plots

The gain plots, shown in Figures 3a,b, uncover the explicit functional dependence of the poles' magnitude and angle on forward gain in the range $10^{-1} \leq k \leq 10^{+3}$. (The breakout point at $s = -0.5$ is not depicted in the gain plots since it occurs at a very low value of gain that does not correspond to the specifications.) The angle gain plot of Figure 3b indicates that two of the closed-loop poles are complex conjugates as depicted in the root locus plot. Also clearly seen is the maximum deviation of the poles from the real axis at a gain of approximately 0.9, corresponding to the pole angles of 250° and 110° or a damping ratio of 0.34. The gain plots indicate that the value of k corresponding to the break-in point and the second break-out point is approximately 12. Also, the range of k that results in purely real poles is so small that it cannot be seen on the scale of Figures 3a,b. Finally, at a gain of approximately 100, the closed-loop system eigenvalues display first order Butterworth asymptotic behavior that is the result of the close proximity of the two poles to the zeros and the one excess pole migrating towards $-\infty$. This is manifested in the constant magnitudes and angles of the two poles approaching the zeros, and the constant angle (180°) and unity slope of the pole magnitude as a function of gain shown in the magnitude gain plot [8].

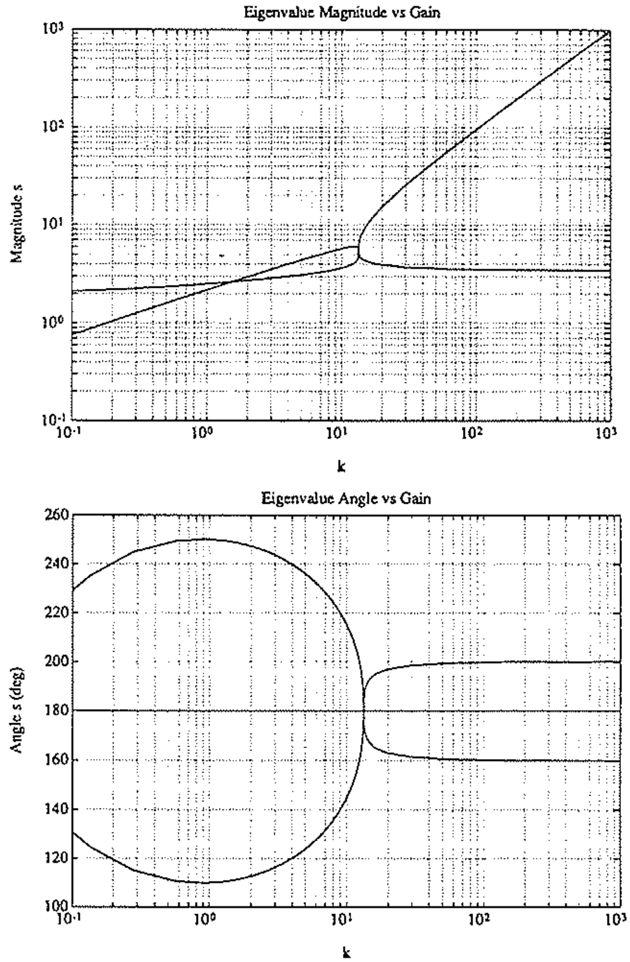

Figure 3a,b. Gain plots for the robot control system of Figure 1.

2.3. Root Sensitivity

The root sensitivity function, S_k, can be determined graphically from the slope of the magnitude gain plot, M_m, and the slope of the angle gain plot, M_a [9]. The relationships are

$$\mathrm{Re}\{S_k\} = M_m \tag{1}$$

$$\mathrm{Im}\{S_k\} = \log_{10}(e) M_a \tag{2}$$

Figures 4a,b are root sensitivity plots for the system of Figure 1.

From the sensitivity plots, it is clear that to meet specification 6, the gain should not be near $k \approx 12$. This corresponds to the break-in point and the second break-out point on the root locus. Clearly, this segment of the root locus is quite sensitive to forward loop gain variations and should be avoided. It should be noted that at high gain, the root sensitivity of the poles migrating towards the zeros asymptotically approach zero; this is indicative of the poles approaching the zeros. Also, for the pole on the real axis the $\mathrm{Re}\{S_k\}$ approaches unity at high gain, which is characteristic of the first order Butterworth pattern. From Figures 4a,b, it is clear that any gain higher than approximately 15 satisfies the root sensitivity requirement.

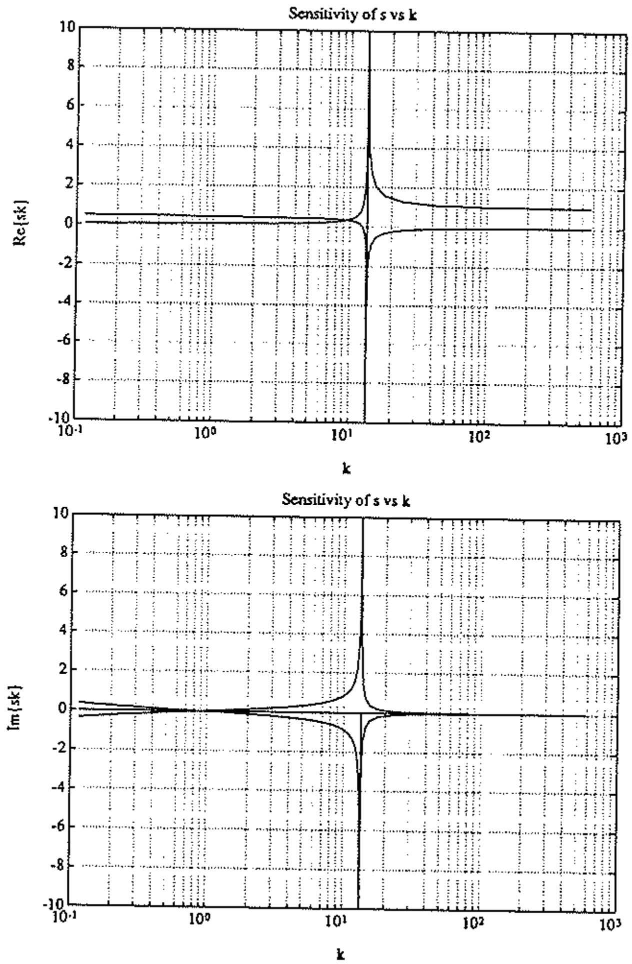

Figure 4a,b. Sensitivity plots for robot control system of Figure 1.

2.4. Parametric Plots

2.4.1. Overshoot

Figure 5 presents the maximum percent overshoot for the closed–loop system as a function of forward gain. The largest peak overshoot occurs at a gain of approximately 0.9; this value results in the lowest damping coefficient as is seen directly in the angle gain plot (Figure 3b). As the gain is increased past unity, the maximum percent overshoot decreases. At a gain of approximately 800 there is no overshoot, indicating that there is effective pole/zero cancellation at $k \approx 800$. However, one may argue that the 1% to 3% overshoot seen at values of $k \approx 100$ corresponds to virtual pole zero cancellation since the closed-loop system behavior is quite close to being first order. By inspection of Figure 5, the gains that satisfy the maximum percent overshoot requirement (specification 1) are approximately $k < 0.18$ and $k > 10$.

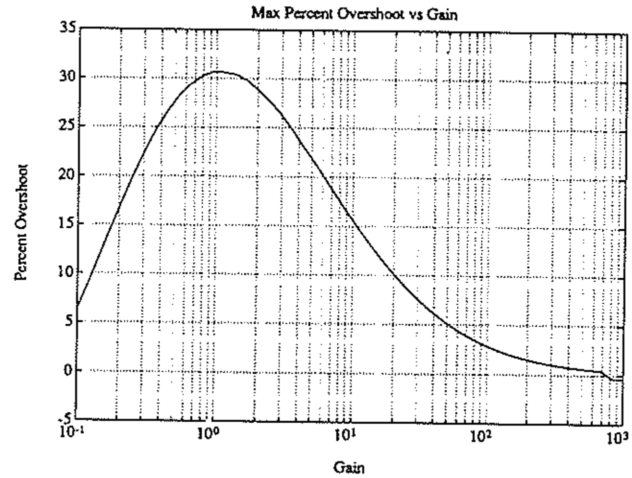

Figure 5. Maximum percent overshoot for the closed–loop system.

2.4.2. Settling Time

In Figure 6 the 2% settling time for the closed–loop system is plotted as a function of gain. The jagged nature of the graph, particularly at low gains, is a result of the definition of the 2% settling time, *i.e.*, it is the time after which the transient response remains within a 2% bound of the steady state step response value. As such, the settling time is not guaranteed to be a continuous function of gain, and may vary substantially depending on the intersection of the transient response and the 2% envelop about the steady state value. As the gain is increased above approximately 0.5, the settling time tends to decrease. From Figure 6, it is clear that to satisfy specification 2 (*i.e.*, 2% settling time in less than 3 seconds), $k > 2.5$.

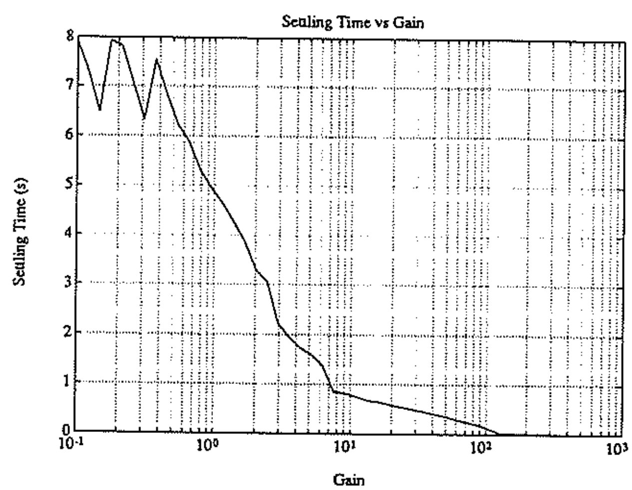

Figure 6. 2% settling time for the closed–loop system.

2.4.3. Rise Time

Figure 7 is a plot of the rise time of the closed loop system as a function of gain. With increasing gain, the rise time is reduced. This is expected since the dominant poles shown on the root locus move further into the right hand plane. At higher values of gain, the two complex poles effectively cancel the zeros, and the third pole on the real axis becomes the dominant pole. It continues to migrate towards $-\infty$ further reducing the rise time. From Figure 7, the range of gain that satisfies specification 3 is $k < 7$.

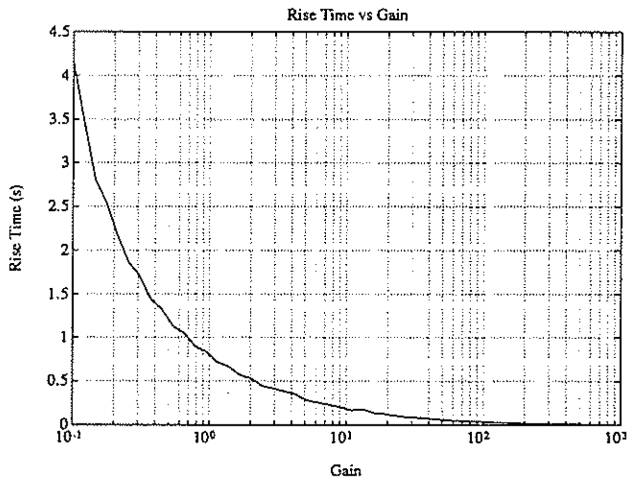

Figure 7. Rise time for the closed–loop system.

2.4.4. Gain and Phase Margin

The closed-loop system, being stable, has infinite gain margin for all positive values of gain. As such, a plot of gain margin *vs.* gain is not shown. (The fact that the system is always stable may be determined from the angle gain plot of Figure 3b; see [10].) Figure 8 is a plot of the phase margin of the closed loop system as a function of gain. It shows that the minimum phase margin is 40° occurring at a gain of approximately 0.9. This coincides with the lowest damping coefficient shown in Figure 3b and the largest percent overshoot shown in Figure 5. As the gain is increased past 0.9, the phase margin asymptotically approaches 90°. At $k \approx 100$, the system phase margin is nearly 90° indicating that the higher gain system may be approximated by a first order system. Again, this is consistent with the other performance measure plots. From Figure 8, gains that satisfy specification 4 are $k < 0.35$ and $k > 2$.

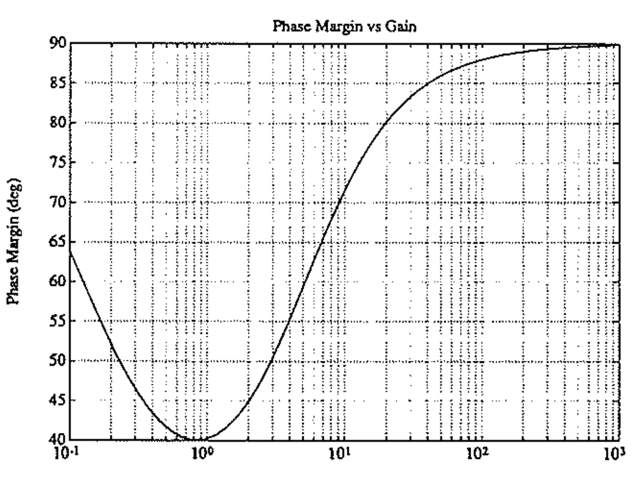

Figure 8. Phase margin for the closed–loop system.

2.5. Design Summary

Table 1 summarizes the results of exploring gain dependence on various system measures, as presented in Figures 2–8. The summary indicates that the closed-loop configuration, with a single forward gain, cannot simultaneously meet all of the requirements. In particular, the rise time specification is in direct conflict with the maximum percent overshoot requirement.

Specification	Figure Number	Range of k
1	5	$k < 0.18, k > 10$
2	6	$k > 2.5$
3	7	$k < 7$
4	8	$k < 0.35, k > 2$
5	NA	$k > 0$
6	3 a,b; 4 a,b	$k < 11, k > 13$

Table 1. Summary of Control System Design and Analysis Plots.

Several options are available to resolve the conflicting specifications including: modifying the specifications, modifying the control configuration and/or modifying the plant configuration to achieve different dynamics. Toward resolving the design conflict, the parametric plots are quite useful. From Figures 5 and 7, it is clear that slight modifications in the requirements enable the proposed control configuration to be acceptable. For example, permitting the rise time to be reduced to 0.2 seconds and the maximum percent overshoot to be increased to 18% yields a viable solution without modifying the plant or controller. This slight relaxation of requirements, if tolerable, will yield acceptable closed-loop system behavior. Again, these tradeoffs are clearly visible from the parametric plots shown in Figures 2-8. Note that the limiting specification, rise time, is used to constrain the power requirement of the actuator motors. Higher power motors are increasingly costly and typically are nonlinear.

Figure 9 is the unit step response of the closed-loop system shown in Figure 1 with a forward gain of 8. It can be verified that the new set of requirements has been met.

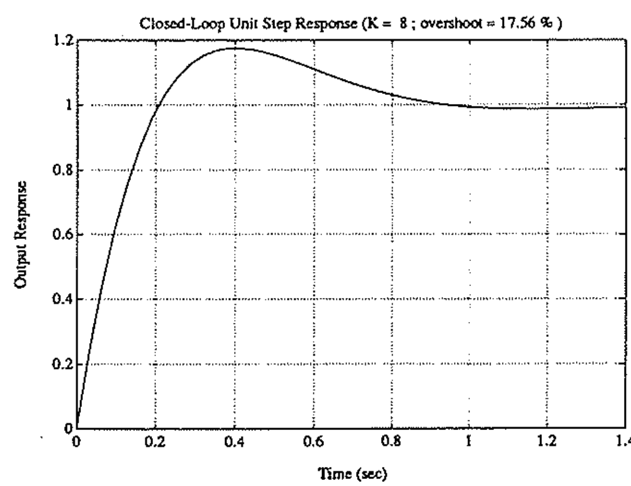

Figure 9. Unit step response of closed-loop system.

3. CONCLUSIONS

A control system usually requires some adjustment so that the various conflicting and demanding specifications can be met. This paper demonstrates that a set of parametric plots, used in a properly orchestrated manner, can aid in the control system design process. The parametric plots suggested here were generated in a matter of minutes with simple commands on microcomputers; they are readily available and recommended tools for the control engineer.

The key to a solid understanding of the design tradeoffs is the ability to view the system parameters from a variety of perspectives simultaneously. Here only a small number of design specifications is considered; if other requirements are introduced, additional parametric plots may be generated. The graphical analyses presented in this paper give the control designer the ability to quickly determine if specifications can be met, and what, if any, tradeoffs are necessary to complete the control design.

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