AN AUTOMATED METHOD FOR CHARACTERIZING CROSS-SECTIONAL PROPERTIES OF COMPLEX SHAPES

Mark L. Nagurka and Wilson C. Hayes
Department of Orthopaedic Surgical Research
127 Johnson Pavilion
University of Pennsylvania
Philadelphia, Pennsylvania 19104

The cross-sectional properties of complex shapes are required in many studies of musculoskeletal biomechanics. To calculate these properties, many previous investigators have used regular geometries such as circles or ellipses to represent biological cross-sections. However, this approach involves a degree of simplification which often does not adequately reflect the complexity of the actual shape. As a result, the subtle changes in cross-sectional properties that can occur in adaptive, biological systems have been inadequately studied. In particular, the mechanisms involved in the stress-related adaptive response of bone have remained obscure.

The purpose of this report is to provide an interactive computer graphics software package for calculating the area cross-sectional properties of irregular, two-dimensional shapes. The program requires input of the boundary coordinates of multiple, planar regions each of which may be multiply connected. The program then calculates the area, centroid, area moments of inertia, principal moments and their orientations. The approach thus allows rapid analysis of the section properties of complex, biological cross-sections and facilitates both the mechanical analysis of musculoskeletal components and the quantification of biological changes in cross-sectional geometry due to aging, metabolic bone disease, or the presence of an internal fixation device. In the following, the basic algorithm used for these calculations is described and the FORTRAN implementation of the technique is outlined. Three examples of the application of the method are then discussed.

The software package is based on a simple algorithm for generating area cross-sectional properties from perimeter coordinates (Wojciechowski, 1976; Hewlett Packard HP-97 ME Pac). The method divides a cross-sectional area into a series of trapezoids (or rectangles) and then adds or subtracts the properties of the elemental areas to determine the composite properties of the total area. This technique replaces integration with the summation of finite elements, and assumes linear segments between consecutive perimeter coordinates.

SLICE is the FORTRAN implementation of this method which was developed and executed on a DEC System 10 at the University of Pennsylvania Medical School Computer Facility. Using a Tektronix 4010 graphics terminal, the program graphically displays the cross-section with superimposed vectors which represent the principal moments of inertia and their orientation. On the same display, section properties are reported to four significant figures. The output variables (and FORTRAN names) are the area (AREA); the x and y coordinates of the centroid (XBAR,YBAR); the moments of inertia about the x and y axes (IX,IY); the product of inertia (IXY); the moments of inertia about the x and y axes translated to the centroid (IXBAR, IYBAR); the product of inertia about the translated axes (IXBYB); the angle between the translated axis and principal axis (PHI); and the moments of inertia about the translated and rotated principal x and y axes (IXBPH, IYBPH). Thus, the cross-section picture and section property information are output on one display "page" (screen), allowing for rapid correlation between changes in cross-section geometries and resulting changes in section properties.

To use SLICE, the cross-section must be located entirely within the first quadrant, that is, with all perimeter coordinates greater than or equal to zero. The perimeter coordinates must be input sequentially in a clockwise path around the cross-section. "Holes" in the cross-section may be deleted by inputting the perimeter points in a counter-clockwise path. SLICE accepts perimeter coordinates of the outer and interior boundaries by direct terminal keyboard input or from files. However, the program can easily be adapted to accept x,y coordinates directly from a table digitizer.

A number of simple geometric shapes of known area properties have been used as test examples to validate the program and to verify its high degree of accuracy. With simple geometric shapes, such as rectangles and triangles, the program attains accuracy to seven significant figures. For execution of both simple test examples and more complex bone cross-sections the program requires approximately 25 seconds of CPU time (at a cost of just over one dollar). Total cost per cross-section run is relatively constant at approximately three dollars.

A simple example of SLICE graphics output appears in Fig. 1. It represents a triangular cross-section with a triangular "hole". Note that the principal directions are represented by vectors superimposed at the centroid, with their lengths proportional to the principal moments of inertia. In this case, the moment of inertia about the principal y axis (IYBPH) is over five times larger than the moment of inertia about the principal x axis (IXBPH).

SLICE has also been used as a basic subroutine in stress analysis programs which calculate stresses based on elementary beam theory for pure bending and torsion. Fig. 2 shows SLICE output for a cross-section of an orthopedic bone reamer which failed in clinical use. In this case, the stress analyses provide both a reasonable explanation of the reamer's failure and a simple means for investigating design changes. In addition to stress analysis applications, SLICE has been incorporated as a subroutine in programs which calculate the mass and mass inertial properties of three-dimensional bodies. These programs require sequential computerized axial tomographic scans of limb segments to directly generate their mass and mass inertial properties.

Cross-sectional changes in bones subject to different conditions have also been studied quantitatively with the program. Using microradiographs provided by Slatis, et.al. (1978), cross-sections of rabbit tibias examined one day to 36 weeks after compression plate implantation were input to the program to monitor cross-sectional changes and to optimize placement of the plate (Fig. 3).

In summary, a simple algorithm for computing the cross-sectional properties of complex geometric shapes has been programmed for automatic data acquisition and analysis. The program can be used to analyze the area properties of multiple, complex biological cross-sections. Specifically, biological changes in cross-sectional geometries can be quantified to better understand changes due to aging, metabolic bone disease, and the presence of fracture fixation devices. Since the approach is highly automated and makes use of extensive graphics output, area property changes can easily be investigated. The program has been completely verified using known cross-sectional geometries and represents an advance over previous methods in ease of use, time, and accuracy.

REFERENCES

1. Slatis, P., E. Karaharju, T. Holmstrom, J. Ahonen, and P. Paavolainen (1978). Structural Changes in Intact Tubular Bone After Application of Rigid Plates with and without Compression. J. Bone and Joint Surg. 60-A: 516-522.

2. Wojciechowski, F. (1976). Properties of Plane Cross Sections. Machine Design (Jan. 22, 1976).

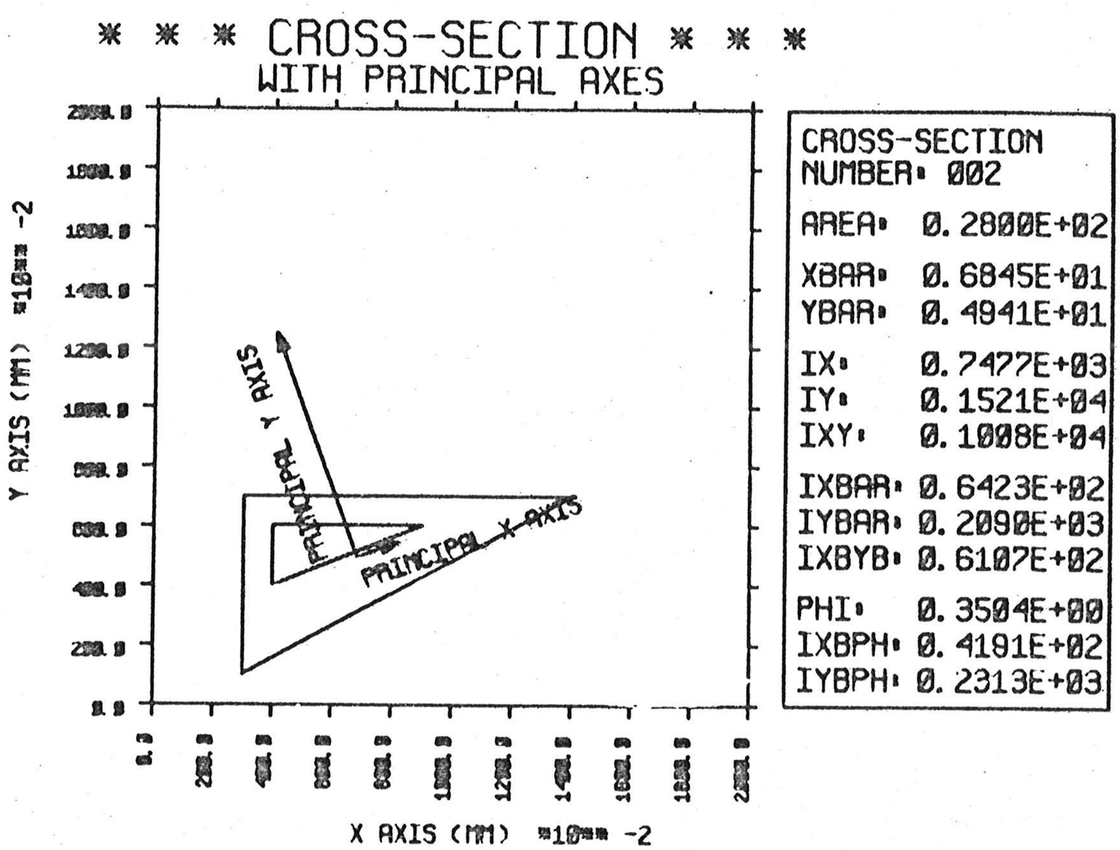

Fig. 1 Test example for program verification.

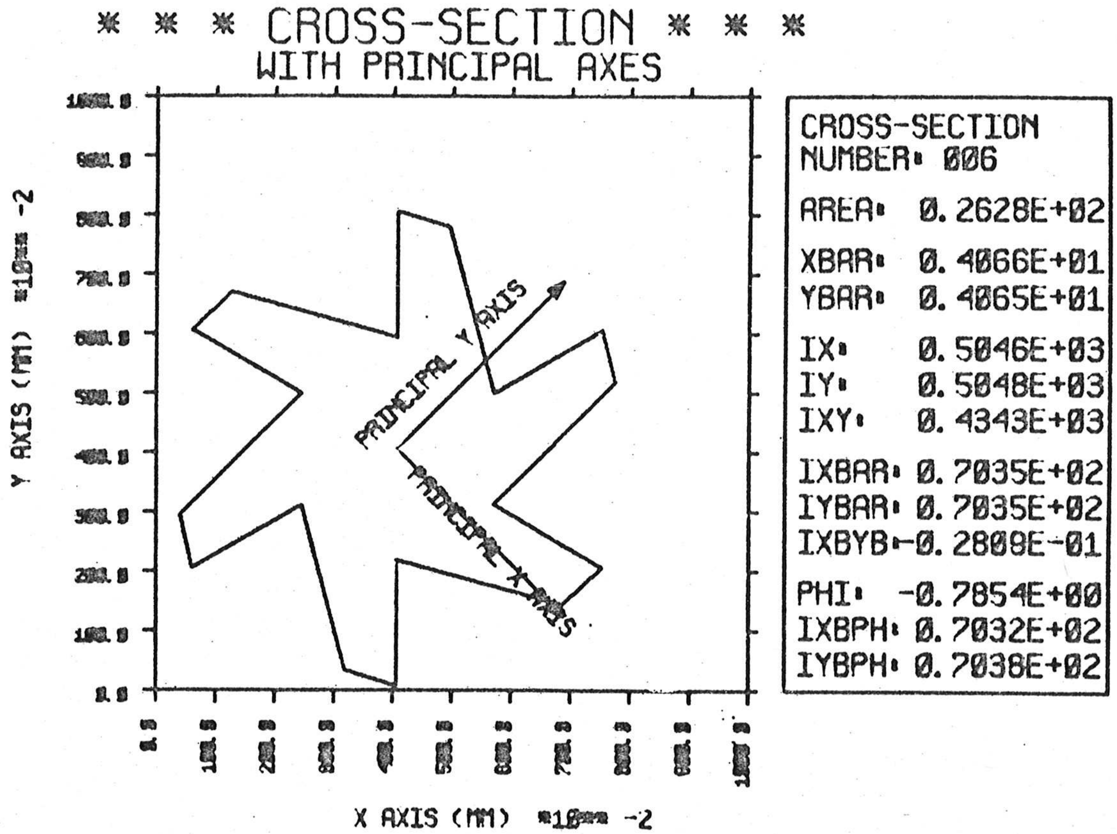

Fig. 2 Distal cross-section of orthopedic reamer.

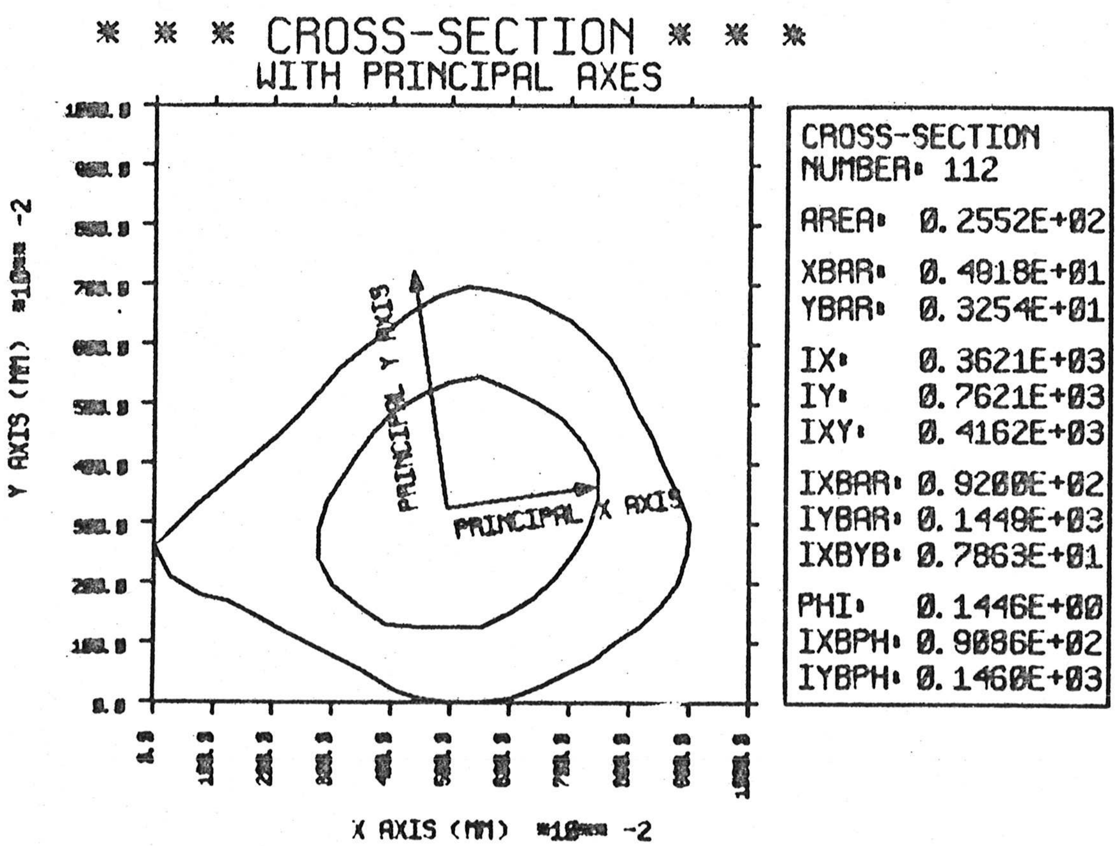

Fig. 3 Cross-section of rabbit tibia after removal of compression plate.