Table 1. Coefficient of side friction.		
Speed (mph)	**Dry Surface**	**Wet Surface**
10	0.45	.38
20	0.40	0.27
30	0.35	0.20
40	0.30	0.16
50	0.26	0.14
55	0.22	0.13
60	0.19	0.12
65	0.18	0.11
70	0.17	0.10
75	0.16	0.09
80	0.15	0.08
85	0.14	0.07

Table 2: Coefficient of forward skidding friction.		
Speed (mph)	**Dry Surface**	**Wet Surface**
10	0.78	0.600
20	0.76	0.400
30	0.74	0.350
40	0.72	0.320
50	0.70	0.305
55	0.67	0.300
60	0.65	0.295
65	0.64	0.290
70	0.63	0.285
75	0.62	0.280
80	0.61	0.275
85	0.60	0.270

$$d_B = \frac{V^2}{30\left[\left(\frac{a}{32.2}\right) \pm G\right]} \quad (3\text{-}3)$$

$$DSD = 1.47Vt + 1.075\frac{V^2}{a} \quad (3\text{-}4)$$

$$R_{\min} = \frac{V^2}{15(0.01e_{\max} + f_{\max})} \quad (3\text{-}8)$$

$$L_r = \frac{(wn_1)\, e_d}{\Delta}(b_w) \quad (3\text{-}23)$$

$$* \; b_w = [1 + 0.5\,(n_1 - 1)]/n_1$$

$$L_t = \frac{e_{NC}}{e_d} L_r \quad (3\text{-}24)$$

$$L = \frac{3.15V^3}{RC} \quad (3\text{-}25)$$

(2.3.7)

$$L = 2\pi R\left(\frac{\Delta}{360}\right) \quad (2.4.1)$$

$$\left.\begin{aligned} \frac{100}{2\pi R} &= \frac{D}{360} \\ D &= \left(\frac{5729.58}{R}\right)^{\circ} \end{aligned}\right\} \quad (2.4.2)$$

$$L = \frac{100\Delta}{D} \quad (2.4.4)$$

E: External distance $= R\left(\sec\frac{\Delta}{2} - 1\right)$ (8)

M: Middle ordinate distance $= R\left(1 - \cos\frac{\Delta}{2}\right)$ (9)

T: Length of tangent $= R\tan\frac{\Delta}{2}$ (10)

L: Length of curvé $= 100\frac{\Delta}{D}$ (11)

LC: Long chord $= 2R\sin\frac{\Delta}{2}$ (12)

$$e = \tan\beta$$

$$e + f_s = \frac{v^2}{gR} \quad (2.4.5)$$

$$e + f_s = \frac{v^2}{15R} \quad (2.4.6)$$

$$R_{\min} = \frac{v^2}{g(e_{\max} + f_{\max})} \quad (2.4.7)$$

$$e_{\text{des}} = \frac{v^2}{gR} - f_s \qquad \text{for } R > R_{\min} \quad (2.4.8)$$

$$M = R\left(1 - \cos\frac{\Delta}{2}\right) \tag{10.3}$$

$$= 1000(1 - \cos 8°19')$$
$$= 10.52'$$

$$E = R \sec\left(\frac{\Delta}{2} - 1\right) \tag{10.4}$$

$$= 1000(\sec 8°19'' - 1)$$
$$= 10.63 \text{ ft}$$

Note: A common mistake made by students first studying circular curves is to determine the station of the EC by adding the T distance to the PI. Although the EC is physically a distance of T from the PI, the stationing (chainage) must reflect the fact that the ℄ no longer goes through the PI. The ℄ now takes the shorter distance (L) from the BC to the EC.

EXAMPLE 10.2

Refer to Figure 10.5. Given

$\Delta = 12°51'$

$R = 400$ m

PI at 0 + 241.782

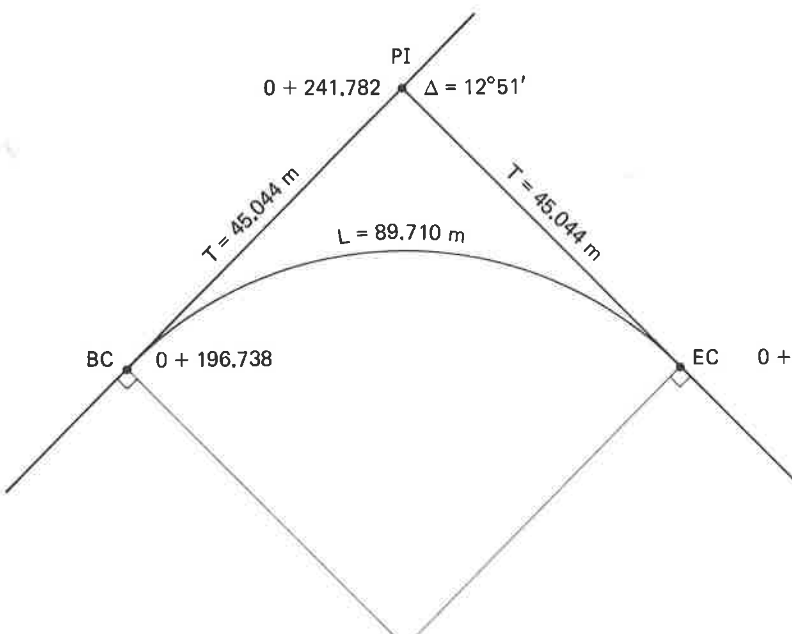


Figure 10.5 Sketch for Example 10.2.

Calculate the station of the BC and EC.

$$T = R \tan \frac{\Delta}{2} \qquad (10\text{-}1) \qquad L = 2\pi R \frac{\Delta}{360} \qquad (10.5)$$

$$= 400 \tan 6°25'30'' \qquad = 2\pi \times 400 \times \frac{12.850}{360}$$

$$= 45.044 \text{ m} \qquad = 89.710 \text{ m}$$

PI at	0 + 241.782
$-T$	45.044
BC =	0 + 196.738
$+L$	89.710
EC =	0 + 286.448

EXAMPLE 10.3

Refer to Figure 10-6. Given

$$\Delta = 11°21'35''$$

PI at 14 + 87.33

$$D = 6°$$

Calculate the station of the BC and EC.

$$R = \frac{5729.58}{D} = 954.93 \text{ ft} \qquad (10.6)$$

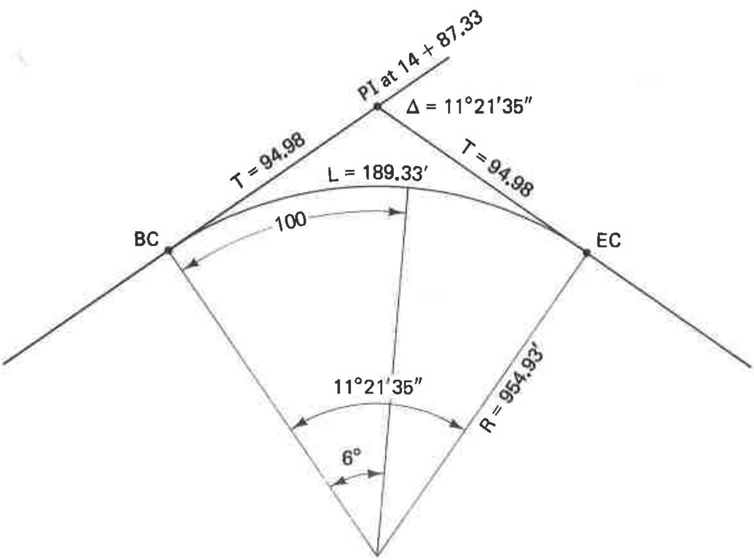


Figure 10.6 Sketch for Example 10.3.

$$T = R \tan \frac{\Delta}{2} = 954.93 \tan 5.679861° \quad (10.1)$$

$$= 94.98 \text{ ft}$$

$$L = 100 \frac{\Delta}{D} = \frac{100 \times 11.359722}{6} \quad (10.7)$$

$$= 189.33 \text{ ft}$$

or

$$L = \frac{2\pi R\Delta}{360} = 2\pi \times \frac{954.93 \times 11.359722}{360} \quad (10.5)$$
$$= 189.33 \text{ ft}$$

PI at	14 + 87.33
$-T$	94.98
BC =	13 + 92.35
$+L$	1 89.33
EC =	15 + 81.68

10.4 CIRCULAR CURVE DEFLECTIONS

The most common method of locating a curve in the field is by deflection angles. Typically, the theodolite is set up at the BC, and the deflection angles are turned from the tangent line (see Figure 10.7).

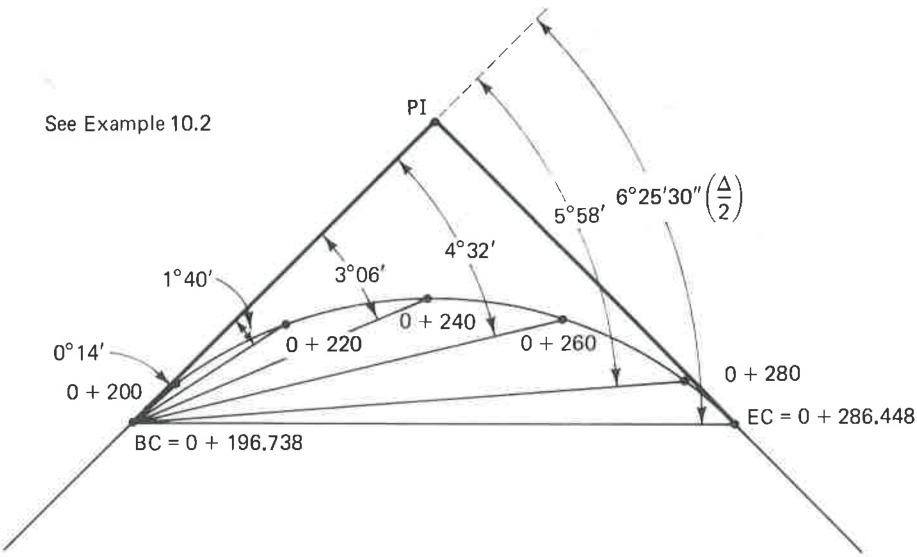


Figure 10.7 Field location for deflection angles.

If we use the data from Example 10.2

$$\text{BC at } 0 + 196.738$$

$$\text{EC at } 0 + 286.448$$

$$\frac{\Delta}{2} = 6°25'30'' = 6.4250°$$

$$L = 89.710$$

$$T = 45.044$$

And if the layout is to proceed at 20-m intervals, the procedure would be as follows. First, compute the deflection angles for the three required arc distances:

$$\text{deflection angle} = \left(\frac{\text{arc}}{\text{L}}\right)\frac{\Delta}{2}$$

1. BC to first even station (0 + 200): ([0 + 200] − [0 + 196.738] = 3.262)

$$\frac{6.4250}{89.710} \times 3.262 = 0.2336° = 0°14'01''$$

2. Even station interval:

$$\frac{6.4250}{89.710} \times 20 = 1.4324° = 1°25'57''$$

3. Last even station (0 + 280) to EC:

$$\frac{6.4250}{89.710} \times 6.448 = 0.4618° = 0°27'42''$$

Second, prepare a list of appropriate stations together with *cumulative* deflection angles.

Stations	Deflection Angle
BC 0 + 196.738	0°00′00″
0 + 200	0°14′01″ + 1°25′57″
0 + 220	1°39′58″ + 1°25′57″
0 + 240	3°05′55″ + 1°25′57″
0 + 260	4°31′52″ + 1°25′57″
0 + 280	5°57′49″ + 0°27′42″
EC 0 + 286.448	6°25′31″ ≈ 6°25′30″ = $\frac{\Delta}{2}$

For most engineering layouts, the deflection angles are rounded to the closest minute or half-minute.

$$E = \frac{AL}{800} \text{ ft} \quad (2.4.11)$$

$$y = 4E\left(\frac{x}{L}\right)^2 \quad (2.4.12)$$

$$X = \frac{LG_1}{G_1 - G_2} \qquad X \geqslant 0 \quad (2.4.13)$$

$$\text{Elevation of } P = \left[\text{elevation of VPC} + \left(\frac{G_1}{100}\right)x\right] + y \quad (2.4.14)$$

Crest vertical curves:

$$L = \frac{|A|S^2}{200(\sqrt{h_1} + \sqrt{h_2})^2} \qquad \text{for } S \leqslant L \quad (2.4.15a)$$

$$L = 2S - \frac{200(\sqrt{h_1} + \sqrt{h_2})^2}{|A|} \qquad \text{for } S \geqslant L \quad (2.4.15b)$$

Sag vertical curves:

$$L = \frac{|A|S^2}{200(h + S \tan \beta)} \qquad \text{for } S \leqslant L \quad (2.4.16a)$$

$$L = 2S - \frac{200(h + S \tan \beta)}{|A|} \qquad \text{for } S \geqslant L \quad (2.4.16b)$$

where

L = length of curve, in ft

S = sight distance, in ft

$|A| = |G_2 - G_1|$, in %

h_1 = height of driver's eyes, in ft

h_2 = height of object, in ft

h = headlight height: approximately 2 ft

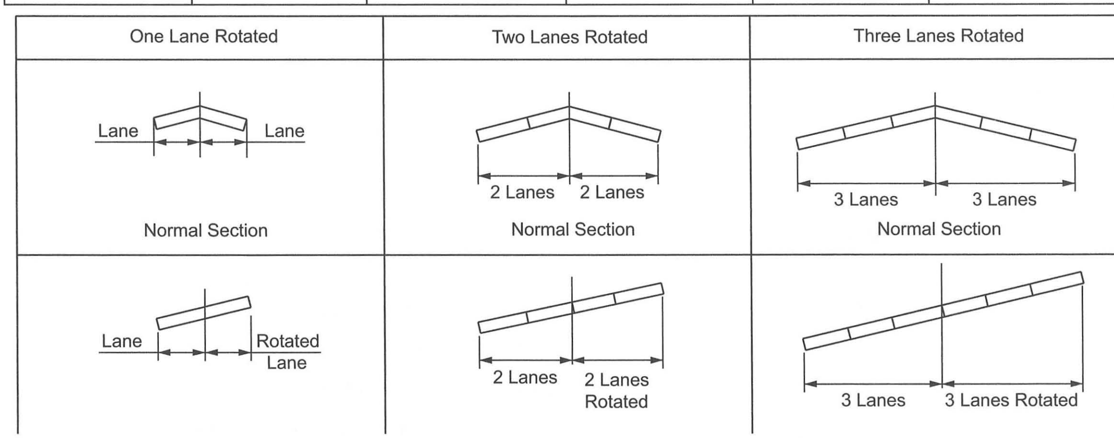
β = beam angle: approximately 1°

A strict application of the maximum relative gradient criterion provides runoff lengths for four-lane undivided roadways that are double those for two-lane roadways; those for six-lane undivided roadways would be tripled. While lengths of this order may be considered desirable, it is often not practical to provide such lengths in design. On a purely empirical basis, it is recommended that minimum superelevation runoff lengths be adjusted downward to avoid excessive lengths for multilane roadways. The recommended adjustment factors are presented in Table 3-16.

The adjustment factors listed in Table 3-16 are directly applicable to undivided streets and highways. Development of runoff for divided highways is discussed in more detail later in the subsection titled, "Axis of Rotation with a Median." The topic of runoff superelevation for turning roadway designs at intersections and through interchanges is discussed in Chapters 9 and 10, respectively.

Table 3-16. Adjustment Factor for Number of Lanes Rotated

Metric			U.S. Customary		
Number of Lanes Rotated, n_1	**Adjustment Factor,*** b_w	**Length Increase Relative to One-Lane Rotated,** $(= n_1 b_w)$	**Number of Lanes Rotated,** n_1	**Adjustment Factor,*** b_w	**Length Increase Relative to One-Lane Rotated,** $(= n_1 b_w)$
1	1.00	1.0	1	1.00	1.0
1.5	0.83	1.25	1.5	0.83	1.25
2	0.75	1.5	2	0.75	1.5
2.5	0.70	1.75	2.5	0.70	1.75
3	0.67	2.0	3	0.67	2.0
3.5	0.64	2.25	3.5	0.64	2.25



* $b_w = [1 + 0.5\,(n_1 - 1)]/n_1$

Typical minimum superelevation runoff lengths are presented in Table 3-17. The lengths shown represent cases where one or two lanes are rotated about a pavement edge. The former case is found on two-lane roadways where the pavement is rotated about the centerline or on one-lane interchange ramps where the pavement rotation is about an edge line. The latter case is found on multilane undivided roadways where each direction is separately rotated about an edge line.

Table 3-17b. Superelevation Runoff L_r (ft) for Horizontal Curves

	U.S. Customary																											
	V_d = 15 mph		V_d = 20 mph		V_d = 25 mph		V_d = 30 mph		V_d = 35 mph		V_d = 40 mph		V_d = 45 mph		V_d = 50 mph		V_d = 55 mph		V_d = 60 mph		V_d = 65 mph		V_d = 70 mph		V_d = 75 mph		V_d = 80 mph	
	Number of Lanes Rotated. Note that 1 lane rotated is typical for a 2-lane highway, 2 lanes rotated is typical for a 4-lane highway, etc. (See Table 3-16.)																											
	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
e (%)	L_r (ft)	L_r (ft)	L_r (ft)	L_r (ft)	L_r (ft)	L_r (ft)	L_r (ft)	L_r (ft)	L_r (ft)	L_r (ft)	L_r (ft)	L_r (ft)	L_r (ft)	L_r (ft)	L_r (ft)	L_r (ft)	L_r (ft)	L_r (ft)	L_r (ft)	L_r (ft)	L_r (ft)	L_r (ft)	L_r (ft)	L_r (ft)	L_r (ft)	L_r (ft)	L_r (ft)	L_r (ft)
1.5	23	35	24	37	26	39	27	41	29	44	31	47	33	50	36	54	38	58	40	60	42	63	45	68	47	71	52	77
2.0	31	46	32	49	34	51	36	55	39	58	41	62	44	67	48	72	51	77	53	80	56	84	60	90	63	95	69	103
2.2	34	51	36	54	38	57	40	60	43	64	46	68	49	73	53	79	56	84	59	88	61	92	66	99	69	104	75	113
2.4	37	55	39	58	41	62	44	65	46	70	50	74	53	80	58	86	61	92	64	96	67	100	72	108	76	114	82	123
2.6	40	60	42	63	45	67	47	71	50	75	54	81	58	87	62	94	66	100	69	104	73	109	78	117	82	123	89	134
2.8	43	65	45	68	48	72	51	76	54	81	58	87	62	93	67	101	71	107	75	112	78	117	84	126	88	133	96	144
3.0	46	69	49	73	51	77	55	82	58	87	62	93	67	100	72	108	77	115	80	120	84	126	90	135	95	142	103	154
3.2	49	74	52	78	55	82	58	87	62	93	66	99	71	107	77	115	82	123	85	128	89	134	96	144	101	152	110	165
3.4	52	78	55	83	58	87	62	93	66	99	70	106	76	113	82	122	87	130	91	136	95	142	102	153	107	161	117	175
3.6	55	83	58	88	62	93	65	98	70	105	74	112	80	120	86	130	92	138	96	144	100	151	108	162	114	171	123	185
3.8	58	88	62	92	65	98	69	104	74	110	79	118	84	127	91	137	97	146	101	152	106	159	114	171	120	180	130	195
4.0	62	92	65	97	69	103	73	109	77	116	83	124	89	133	96	144	102	153	107	160	112	167	120	180	126	189	137	206
4.2	65	97	68	102	72	108	76	115	81	122	87	130	93	140	101	151	107	161	112	168	117	176	126	189	133	199	144	216
4.4	68	102	71	107	75	113	80	120	85	128	91	137	98	147	106	158	112	169	117	176	123	184	132	198	139	208	151	226
4.6	71	106	75	112	79	118	84	125	89	134	95	143	102	153	110	166	117	176	123	184	128	193	138	207	145	218	158	237
4.8	74	111	78	117	82	123	87	131	93	139	99	149	107	160	115	173	123	184	128	192	134	201	144	216	152	227	165	247
5.0	77	115	81	122	86	129	91	136	97	145	103	155	111	167	120	180	128	191	133	200	140	209	150	225	158	237	171	257
5.2	80	120	84	126	89	134	95	142	101	151	108	161	116	173	125	187	133	199	139	208	145	218	156	234	164	246	178	267
5.4	83	125	88	131	93	139	98	147	105	157	112	168	120	180	130	194	138	207	144	216	151	226	162	243	171	256	185	278
5.6	86	129	91	136	96	144	102	153	108	163	116	174	124	187	134	202	143	214	149	224	156	234	168	252	177	265	192	288
5.8	89	134	94	141	99	149	105	158	112	168	120	180	129	193	139	209	148	222	155	232	162	243	174	261	183	275	199	298
6.0	92	138	97	146	103	154	109	164	116	174	124	186	133	200	144	216	153	230	160	240	167	251	180	270	189	284	206	309
6.2	95	143	101	151	106	159	113	169	120	180	128	192	138	207	149	223	158	237	165	248	173	260	186	279	196	294	213	319
6.4	98	148	104	156	110	165	116	175	124	186	132	199	142	213	154	230	163	245	171	256	179	268	192	288	202	303	219	329
6.6	102	152	107	161	113	170	120	180	128	192	137	205	147	220	158	238	169	253	176	264	184	276	198	297	208	313	226	339
6.8	105	157	110	165	117	175	124	185	132	197	141	211	151	227	163	245	174	260	181	272	190	285	204	306	215	322	233	350
7.0	108	162	114	170	120	180	127	191	135	203	145	217	156	233	168	252	179	268	187	280	195	293	210	315	221	332	240	360
7.2	111	166	117	175	123	185	131	196	139	209	149	223	160	240	173	259	184	276	192	288	201	301	216	324	227	341	247	370
7.4	114	171	120	180	127	190	135	202	143	215	153	230	164	247	178	266	189	283	197	296	207	310	222	333	234	351	254	381
7.6	117	175	123	185	130	195	138	207	147	221	157	236	169	253	182	274	194	291	203	304	212	318	228	342	240	360	261	391
7.8	120	180	126	190	134	201	142	213	151	226	161	242	173	260	187	281	199	299	208	312	218	327	234	351	246	369	267	401
8.0	123	185	130	195	137	206	145	218	155	232	166	248	178	267	192	288	204	306	213	320	223	335	240	360	253	379	274	411
8.2	126	189	133	199	141	211	149	224	159	238	170	254	182	273	197	295	209	314	219	328	229	343	246	369	259	388	281	422
8.4	129	194	136	204	144	216	153	229	163	244	174	261	187	280	202	302	214	322	224	336	234	352	252	378	265	398	288	432
8.6	132	198	139	209	147	221	156	235	166	250	178	267	191	287	206	310	220	329	229	344	240	360	258	387	272	407	295	442
8.8	135	203	143	214	151	226	160	240	170	255	182	273	196	293	211	317	225	337	235	352	246	368	264	396	278	417	302	453
9.0	138	208	146	219	154	231	164	245	174	261	186	279	200	300	216	324	230	345	240	360	251	377	270	405	284	426	309	463
9.2	142	212	149	224	158	237	167	251	178	267	190	286	204	307	221	331	235	352	245	368	257	385	276	414	291	436	315	473
9.4	145	217	152	229	161	242	171	256	182	273	194	292	209	313	226	338	240	360	251	376	262	393	282	423	297	445	322	483
9.6	148	222	156	234	165	247	175	262	186	279	199	298	213	320	230	346	245	368	256	384	268	402	288	432	303	455	329	494
9.8	151	226	159	238	168	252	178	267	190	285	203	304	218	327	235	353	250	375	261	392	273	410	294	441	309	464	336	504
10.0	154	231	162	243	171	257	182	273	194	290	207	310	222	333	240	360	255	383	267	400	279	419	300	450	316	474	343	514
10.2	157	235	165	248	175	262	185	278	197	296	211	317	227	340	245	367	260	391	272	408	285	427	306	459	322	483	350	525
10.4	160	240	169	253	178	267	189	284	201	302	215	323	231	347	250	374	266	398	277	416	290	435	312	468	328	493	357	535
10.6	163	245	172	258	182	273	193	289	205	308	219	329	236	353	254	382	271	406	283	424	296	444	318	477	335	502	363	545
10.8	166	249	175	263	185	278	196	295	209	314	223	335	240	360	259	389	276	414	288	432	301	452	324	486	341	512	370	555
11.0	169	254	178	268	189	283	200	300	213	319	228	341	244	367	264	396	281	421	293	440	307	460	330	495	347	521	377	566
11.2	172	258	182	272	192	288	204	305	217	325	232	348	249	373	269	403	286	429	299	448	313	469	336	504	354	531	384	576
11.4	175	263	185	277	195	293	207	311	221	331	236	354	253	380	274	410	291	437	304	456	318	477	342	513	360	540	391	586
11.6	178	268	188	282	199	298	211	316	225	337	240	360	258	387	278	418	296	444	309	464	324	486	348	522	366	549	398	597
11.8	182	272	191	287	202	303	215	322	228	343	244	366	262	393	283	425	301	452	315	472	329	494	354	531	373	559	405	607
12.0	185	277	195	292	206	309	218	327	232	348	248	372	267	400	288	432	306	460	320	480	335	502	360	540	379	568	411	617

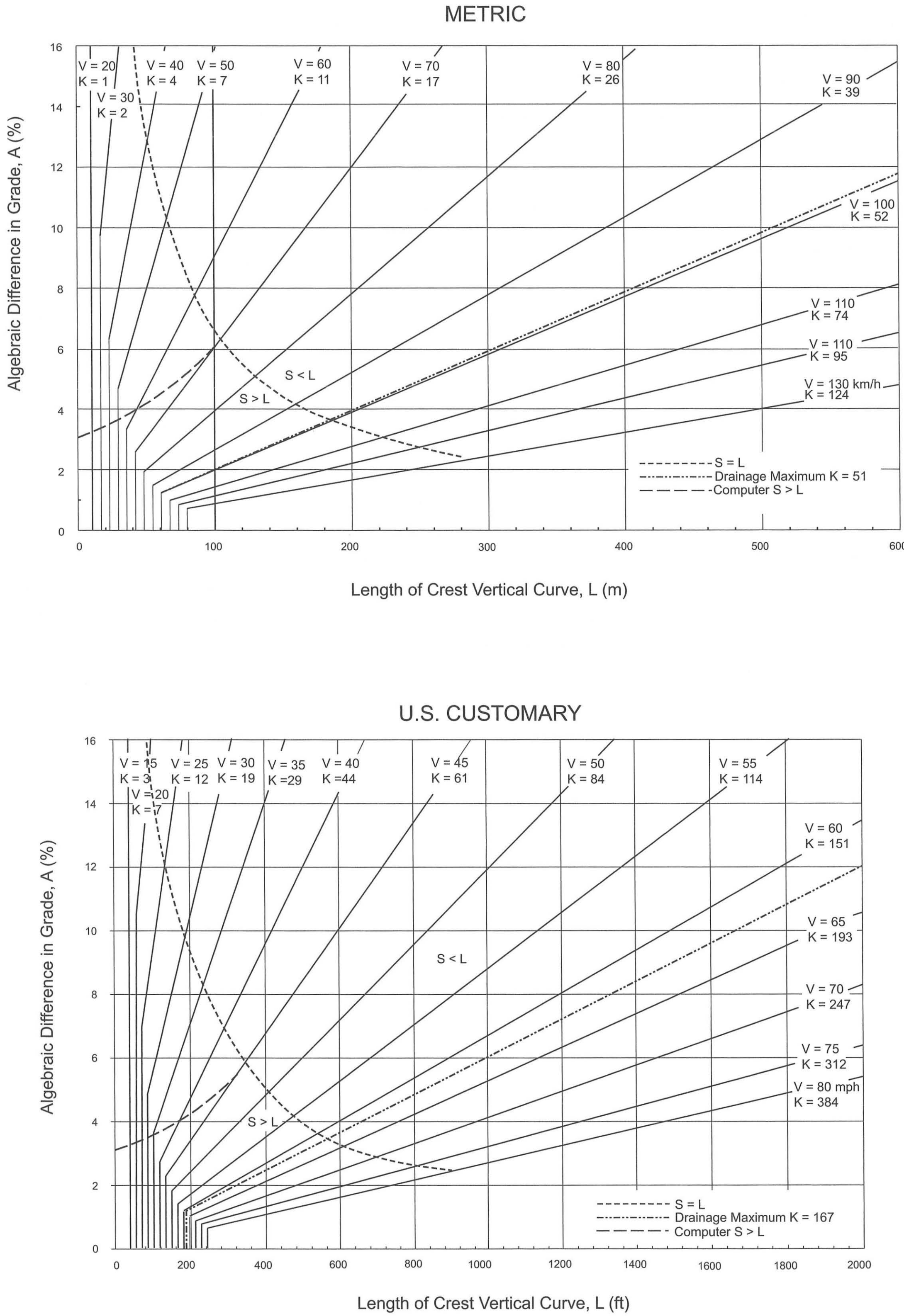


Figure 3-43. Design Controls for Crest Vertical Curves—Open Road Conditions

Table 3-34. Design Controls for Crest Vertical Curves Based on Stopping Sight Distance

Metric				U.S. Customary			
Design Speed (km/h)	Stopping Sight Distance (m)	Rate of Vertical Curvature, K^a		Design Speed (mph)	Stopping Sight Distance (ft)	Rate of Vertical Curvature, K^a	
		Calculated	Design			Calculated	Design
20	20	0.6	1	15	80	3.0	3
30	35	1.9	2	20	115	6.1	7
40	50	3.8	4	25	155	11.1	12
50	65	6.4	7	30	200	18.5	19
60	85	11.0	11	35	250	29.0	29
70	105	16.8	17	40	305	43.1	44
80	130	25.7	26	45	360	60.1	61
90	160	38.9	39	50	425	83.7	84
100	185	52.0	52	55	495	113.5	114
110	220	73.6	74	60	570	150.6	151
120	250	95.0	95	65	645	192.8	193
130	285	123.4	124	70	730	246.9	247
				75	820	311.6	312
				80	910	383.7	384

[a] Rate of vertical curvature, K, is the length of curve per percent algebraic difference in intersecting grades (A), $K = L/A$.

The values of K derived above when S is less than L also can be used without significant error where S is greater than L. As shown in Figure 3-42, extension of the diagonal lines to meet the vertical lines for minimum lengths of vertical curves results in appreciable differences from the theoretical only where A is small and little or no additional cost is involved in obtaining longer vertical curves.

For night driving on highways without lighting, the length of visible roadway is that roadway that is directly illuminated by the headlights of the vehicle. For certain conditions, the minimum stopping sight distance values used for design exceed the length of visible roadway. First, vehicle headlights have limitations on the distance over which they can project the light intensity levels that are needed for visibility. When headlights are operated on low beams, the reduced candlepower at the source plus the downward projection angle significantly restrict the length of visible roadway surface. Thus, particularly for high-speed conditions, stopping sight distance values exceed road-surface visibility distances afforded by the low-beam headlights regardless of whether the roadway profile is level or curving vertically. Second, for crest vertical curves, the area forward of the headlight beam's point of tangency with the roadway surface is shadowed and receives only indirect illumination.

Since the headlight mounting height (typically about 0.60 m [2.00 ft]) is lower than the driver eye height used for design (1.08 m [3.50 ft]), the sight distance to an illuminated object is controlled by the height of the vehicle headlights rather than by the direct line of sight. Any object within the shadow zone must be high enough to extend into the headlight beam to be directly illuminated. On the basis of Equation 3-41, the bottom of the headlight beam is about 0.40 m [1.30 ft] above the roadway at a distance ahead of the vehicle equal to the stopping sight distance. Although the vehicle headlight system does limit roadway

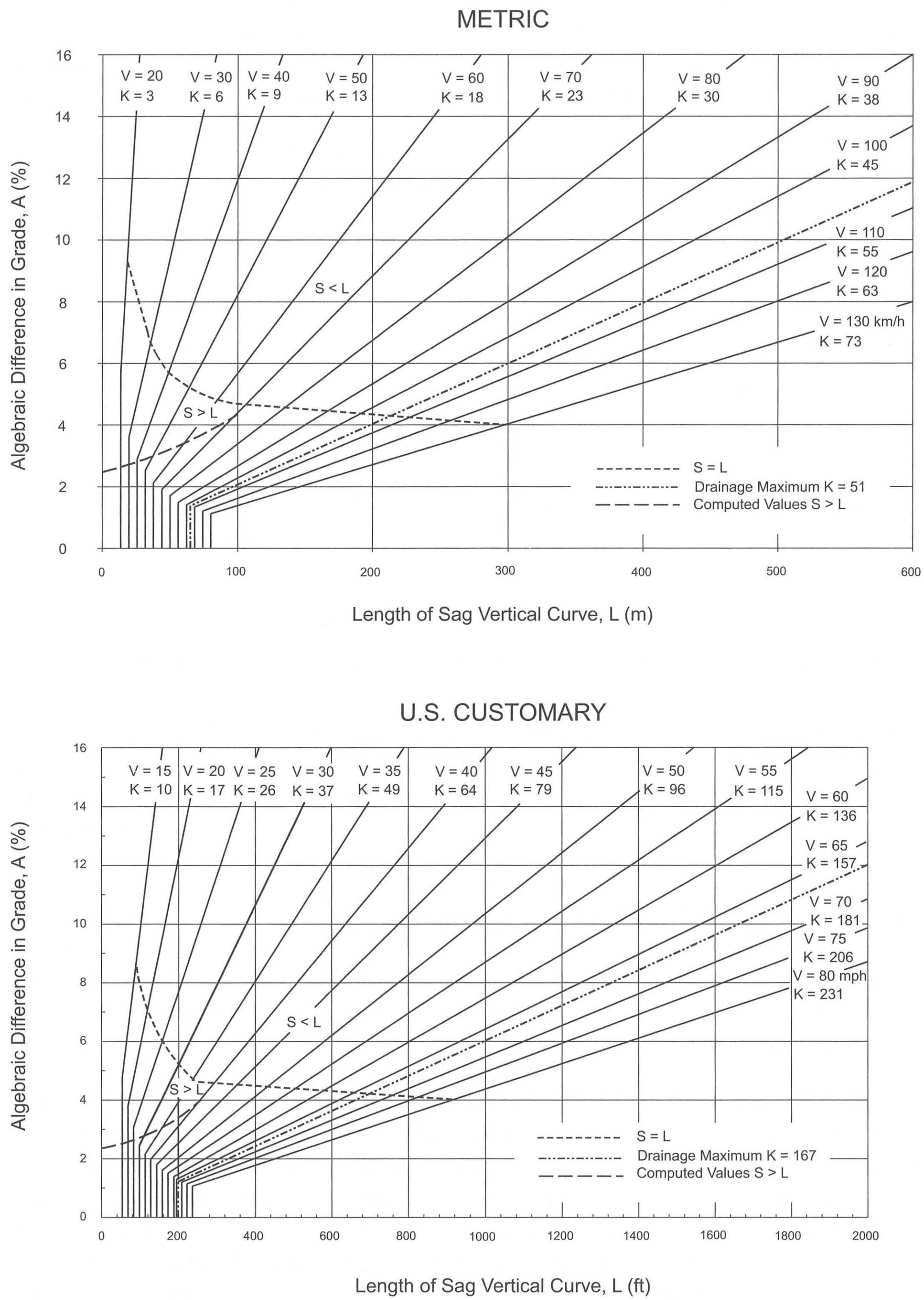


Figure 3-44. Design Controls for Sag Vertical Curves—Open Road Conditions

The effect on passenger comfort of the change in vertical direction is greater on sag than on crest vertical curves because gravitational and centripetal forces are combining rather than opposing forces. Comfort wherever practical, but special attention to drainage should be exercised where values of K in excess of 51 m [167 ft] per percent change in grade are used.

Minimum lengths of vertical curves for flat gradients also are recognized for sag conditions. The values determined for crest conditions appear to be generally suitable for sags. Lengths of sag vertical curves, shown as vertical lines in Figure 3-44, are equal to 0.6 times the design speed in km/h [three times the design speed in mph].

Sag vertical curves shorter than the lengths computed from Table 3-36 may be justified for economic reasons in cases where an existing feature, such as a structure not ready for replacement, controls the vertical profile. In certain cases, ramps may also be designed with shorter sag vertical curves. Fixed-source lighting is desirable in such cases. For street design, some engineers accept design of a sag or crest where A is about 1 percent or less without a length of calculated vertical curve. However, field modifications during construction usually result in constructing the equivalent to a vertical curve, even if short.

Table 3-36. Design Controls for Sag Vertical Curves

Metric				U.S. Customary			
Design Speed (km/h)	Stopping Sight Distance (m)	Rate of Vertical Curvature, K^a		Design Speed (mph)	Stopping Sight Distance (ft)	Rate of Vertical Curvature, K^a	
		Calculated	Design			Calculated	Design
20	20	2.1	3	15	80	9.4	10
30	35	5.1	6	20	115	16.5	17
40	50	8.5	9	25	155	25.5	26
50	65	12.2	13	30	200	36.4	37
60	85	17.3	18	35	250	49.0	49
70	105	22.6	23	40	305	63.4	64
80	130	29.4	30	45	360	78.1	79
90	160	37.6	38	50	425	95.7	96
100	185	44.6	45	55	495	114.9	115
110	220	54.4	55	60	570	135.7	136
120	250	62.8	63	65	645	156.5	157
130	285	72.7	73	70	730	180.3	181
				75	820	205.6	206
				80	910	231.0	231

[a] Rate of vertical curvature, K, is the length of curve (m) per percent algebraic difference intersecting grades (A), $K = L/A$.

Sight Distance at Undercrossings

Sight distance on the highway through a grade separation should be at least as long as the minimum stopping sight distance and preferably longer. Design of the vertical alignment is the same as at any other point on the highway except in some cases of sag vertical curves underpassing a structure as illustrated in Figure 3-45. While not a frequent concern, the structure fascia may cut the line of sight and limit the sight distance to less than otherwise is attainable. It is generally practical to provide the minimum length of sag vertical curve at grade separation structures, and even where the recommended grades are exceeded,