KINEMATIC STEERING CONTROL OF RAIL VEHICLES

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ABSTRACT

Rail vehicles with forced-steered trucks employ passive linkages between the wheelsets and carpody that sense track curvature and steer the wheelsets accordingly. The linkage dimensions, which are related to a steering gain, are typically selected to kinematically align the wheelsets radially. This paper addresses the effect of the steering gain on the curving performance of a forced-steered truck operating at high speed. The results show that the steering gain corresponding to kinematic radial alignment, called the pure rolling line steering gain, understeers the wheelsets during moderate and tight curve negotiation. For these curving conditions, steering gains slightly larger than the pure rolling line gain offer improved curving performance in terms of decreased wheelset misalignments.

1. INTRODUCTION

Rail vehicles are often required to negotiate tight curves, especially in urban transit systems. During curving negotiation, the performance of conventional vehicles generally suffers from accelerated wear at the wheels and rails, increased fuel consumption, enhanced danger of derailment, and objectionable noise generation. These problems are associated with large wheel/rail contact forces which develop due to angular misalignments of the wheelsets of stiff trucks on curved track. To alleviate these problems, vehicles designed to control wheelset angles and wheel/rail forces have been proposed [1-10].

To improve curving performance, vehicles with trucks with softened suspension stiffnesses have been considered. However, a design tradeoff exists since softening the suspension system reduces the lateral stability of the vehicle and thereby increases the tendency to hunt on straight track. Vehicles with trucks which incorporate innovative suspension systems have been designed to "steer" their wheelsets into radial alignment without degrading dynamic stability. In the forced-steered truck design, (passive) linkages directly interconnect the wheelsets as well as connect the wheelsets to the carpody. In addition to providing the requisite stiffness for stability, the carpody-wheelset linkages measure the track curvature (by sensing the relative yaw angle that develops between the carpody and truck frame) and steer the wheelsets into radial alignment accordingly. For negotiation of moderate and sharp curves, studies have shown that significant improvements in curving performance are possible by employing forced-steered trucks in comparison to conventional trucks and in comparison to self-steered trucks, which only use direct interconnections between the wheelsets and which rely on friction forces for steering [11, 12].

This paper investigates the steady-state curving performance of a forced-steered truck and shows that it is a strong function of the stiffness and dimensions of the forced-steering linkages, the wheel/rail profile, and the track curvature. In particular, this paper suggests that the selection of the steering linkage dimensions based solely on kinematic arguments can result in imperfect steering of the wheelsets.

2. STUDY METHODOLOGY

In this paper, the curving performance is represented by two indices: (i) the contact work due to friction at the flanging wheel (per unit distance traversed), which is a measure of wheel/rail wear [13], and (ii) the leading wheelset angle of attack (or yaw angle with respect to radial alignment), which is a measure of wheelset misalignment.

2.1 Forced-Steered Truck Model

The forced-steered truck model selected for study is shown schematically in Figure 1. In the model, \( k_x \) represents the interaxle shear stiffness due to the direct steering arm interconnection between the wheelsets; \( k_y \) represents the stiffness of the forced-steering linkages and is shown as a stiffness which connects eccentrically a wheelset steering arm to the (carpody) bolster. Not shown in Figure 1 is the primary suspension system, modeled as springs in the longitudinal and lateral directions with stiffnesses \( k_{px} \) and \( k_{py} \) respectively, at each journal bearing.

It should be mentioned that forced-steering actuation can be achieved using different linkage arrangements other than those shown in Figure 1 [1, 2, 11, 12]. Relative performance, design, and manufacturing benefits of specific forced-steered truck configurations are not addressed in this paper.
Detailed studies of alternate configurations represent an important area of future research.

For the model in Figure 1, the effect of the forced-steering action can be represented by a geometric yaw offset, $\Delta \Psi$, in series with an effective interaxle bending stiffness, $k_{b2}$, that produces a forced-steering moment, $\Delta M$, acting on the wheelsets.

$$\Delta M = k_{b2} \Delta \Psi$$  \hspace{1cm} (1)

It can be shown that in equilibrium the effective interaxle bending stiffness, $k_{b2}$, is related to the steering linkage stiffness, $k_{b2}'$, by

$$k_{b2} = \frac{(b - l_1)^2 k_{b2}'}{(4 k_{py} + k_{fs})}$$  \hspace{1cm} (2)

where $k_{b2}'$, as described above, is the primary lateral stiffness. The geometric offset, $\Delta \Psi$, is controlled by the steering law

$$\Delta \Psi = \pm 2 G \left( \frac{Y_{w1} - Y_{w2} - \Psi_c}{2b} \right)$$

$$+ 2 \left( \frac{G + 1}{b} \right) \left( \frac{Y_{w1} + Y_{w2} - \Psi_c}{2} \right)$$  \hspace{1cm} (3)

where $Y_{w1}$, $Y_{w2}$, and $\Psi_c$ are the lateral displacements of the leading wheelset, trailing wheelset, and truck, respectively; $\Psi_c$ is the yaw angle of the carbody; $2b$ is the truck wheelbase; and $G$ is the steering action gain regulated by

$$G = \frac{t_1}{b - l_1}$$  \hspace{1cm} (4)

In equation (3), the notation $\pm$ implies $+$ for the front truck and $-$ for the rear truck. Theoretically, the gain which makes the wheelsets track the pure rolling line ensuring perfect radial alignment of the wheelsets is

$$G_{rl} = \frac{b}{l}$$  \hspace{1cm} (5)

where $G_{rl}$ is the pure rolling line steering gain and $l$ is half the distance between the truck centers [2].

### 2.2 Analytical and Numerical Methods

A methodology has been developed to predict the steady-state behavior on constant radius track of a rail vehicle model [11, 12]. The vehicle model consists of two forced-steered truck models connected to a carbody by secondary suspension elements and forced-steering linkages. The analysis accounts for nonlinear wheel/rail profile geometry, wheel/rail friction force saturation, and nonlinear suspension components. In addition, the analysis considers the important case of two-point wheel/rail contact which occurs with many common wheel profiles during curving.

The performance of the model is evaluated by solving the steady-state curving equations of motion. These equations are a set of simultaneous, nonlinear, algebraic equations which can be written as

$$K(X) \cdot X = B(X)$$  \hspace{1cm} (6)

where the matrix product $K(X) \cdot X$ represents a vector of all internal suspension forces and moments and $B(X)$ represents a vector of all external forces and moments. The matrix product is composed of a displacement state vector, $X$, and a nonlinear stiffness matrix, $K(X)$, due to nonlinear primary and secondary suspension components. Equation (6) is solved using a combined Newton-Raphson and steepest-descent method [14], which has proven to be a very robust solution technique.

### 3. PERFORMANCE STUDIES

In this section, the results of parametric studies that investigate the steady-state curving performance of the forced-steered rail vehicle model, operating at balance speed, are presented. The dominant parameter influencing the curving
performance is the track curvature, which constitutes an input to the wheelsets. Many urban transit systems include sections of track with steep curves. Some systems have curves as high as 30°, although common transit curves are less than 7.5°. In general, yard curves are greater than 7.5°, and often restraining rails are present. In the curving performance studies, four track curvatures have been selected: 2.5°, 5°, 10°, and 20° curves. The 2.5° and 5° curves represent shallow curves of 2280 ft (700 m) and 1150 ft (350 m) radii, respectively, the 10° curve represents a moderate curve of 575 ft (175 m), and the 20° curve represents a steep curve of 288 ft (88 m) radius. The 10° curve is assumed to be a representative transit curve which typically may be encountered.

In addition to track curvature, the principal parameters influencing the curving performance are forced-steering linkage stiffness, steering linkage dimensions (or, equivalently, steering gain) and wheel/rail profile. The effects of these parameters are discussed below.

3.1 Performance as a Function of Stiffness

The curving performance of a forced-steered truck is influenced by the stiffness of the forced-steering linkages (or, equivalently, by the effective interaxle bending stiffness, \( k_{px} \)). In this section, the forced-steered truck model with the steering gain set to the pure rolling steering gain, \( G_{p} \), is studied. The gain \( G_{p} \) is the appropriate gain required for the wheelsets to track the pure rolling line and achieve radial alignment, based on kinematic arguments which assume rigid steering linkages. In the following section, the effect of the steering gain on the curving performance is discussed.

The curving (and stability) properties of a forced-steered truck are not unique since different combinations of primary longitudinal stiffness, \( k_{px} \), and effective interaxle bending stiffness, \( k_{px} \), can result in the same total effective truck bending stiffness. In this paper, a forced-steered truck with a soft \( k_{px} \) of 7.0x10^5 lb/ft (1.0x10^6 N/m) is studied. The primary longitudinal stiffness is assumed constant and in the stiffness studies, the interaxle bending stiffness (or, equivalently, the linkage stiffness) is the design parameter. It should be noted that the primary longitudinal stiffness which has been selected is softer than that typically used in current transit trucks. There is, however, significant interest in designing trucks with reduced primary suspension stiffness to improve curving performance. In these trucks, the interwheelset and wheelset-carbody linkages provide the requisite stiffness for stability.

The curving performance in terms of flanging wheel work as a function of effective interaxle

\[ D = \frac{360}{\sin \left( \frac{1}{50} \right)} \text{ ft} \]

\[ \text{with } R \text{ in ft.} \]

bending stiffness, \( k_{px} \), is shown in Figure 2 for a forced-steered truck with new AAR wheels, negotiating four different track curvatures. In the shallow curves, the wheelsets of a truck with soft \( k_{px} \) maintain near radial orientations due to longitudinal friction forces which overcome the small yaw bending resistance. As the truck with soft \( k_{px} \), negotiates steeper curves, sustained flange contact occurs at the leading outer wheel. The leading wheelset cannot track the pure rolling line (since it exceeds the flange clearance), resulting in large longitudinal friction forces at the wheels. To satisfy equilibrium, the wheelset adopts a positive yaw angle of attack and the work at the flanging wheel becomes large. For a truck with stiffer \( k_{px} \), negotiating all degree curves, the wheelsets are more restrained in yaw and forced-steering action is used to "force" the wheelsets toward radial alignment. Interestingly, the flanging wheel work rises slightly with stiffer \( k_{px} \). As will be shown below, the pure rolling line gain, which was selected to kinematically align the wheelsets radially, slightly understeers the leading wheelset. As \( k_{px} \), stiffens, the leading wheelset adopts a slightly increasing positive angle of attack due to insufficient steering action and the flanging wheel work increases.

![Figure 2: Work at Flanging Wheel vs. Interaxle Bending Stiffness of a Forced-Steered Truck with New AAR Wheels Negotiating 2.5°, 5°, 10°, and 20° Curves.](image)

3.2 Performance as a Function of Steering Gain

The dimensions of the steering linkages govern the steering gain and play a major role in determining the curving performance. In this section, the effect of steering gain is explored for a truck with soft primary longitudinal stiffness \( \left( k_{px} = 7.0x10^5 \text{ lb/ft} = 1.0x10^6 \text{ N/m} \right) \) and stiff effective interaxle bending stiffness \( \left( k_{px} = 1.0x10^7 \text{ ft-lb/ft} = 1.4x10^4 \text{ N-m/ft} \right) \). The truck is operating with two different wheel profiles: new AAR wheels, for which two-point contact occurs at the flanging wheel, and Heumann wheels, for which single-point wheel/rail contact occurs.

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The effect of the steering gain on the leading wheelset angle of attack is shown in Figures 3 and 4 for a truck negotiating 10° and 20° curves, respectively. At \( G = 0 \), no forced-steering action occurs and the truck behaves like a self-steering truck (with stiffness \( h_b \)). As the steering gain increases, the leading wheelset is forced toward radial alignment. At a certain gain, the angle of attack vanishes and perfect steering is achieved. Further increases in steering gain result in oversteering of the wheelset away from radial alignment.

The steering gain associated with perfect radial alignment is higher than the pure rolling line gain for forced-steered trucks with new AAR and Heumann wheels for negotiation of moderate 10° and tight 20° curves. In particular, for the truck studied, the pure rolling line gain is 0.158. For negotiation of 10° curves, the gain for zero angle of attack is 0.195 for the truck with new AAR wheels and 0.175 for the truck with Heumann wheels. For negotiation of 20° curves, the gain for perfect steering is 0.205 for the truck with new AAR wheels and 0.180 for the truck with Heumann wheels. The gain for perfect steering is closer to the pure rolling line gain for negotiation of the 10° curve than the 20° curve for trucks with both new AAR and Heumann wheels. As shallower curves are traversed, the flange forces are smaller and the pure rolling line gain results in improved steering (i.e., decreased understeering).

Figures 3 and 4 show that for a forced-steered truck with a stiff steering linkage stiffness and a pure rolling line steering gain, the leading wheelset adopts slight positive misalignments due to understeering. The understeering is more pronounced in the truck with new AAR wheels than in the truck with Heumann wheels negotiating 10° and 20° curves. In comparison to the truck with Heumann wheels, for which only single point wheelrail contact occurs, two-point contact develops at the leading outer wheel of the truck with new AAR wheels and a smaller "restoring" moment is available to help steer the leading wheelset [11]. Thus, at the pure rolling line gain, a larger wheelset angle of attack occurs for the truck with new AAR wheels than for the truck with Heumann wheels for negotiation of both moderate and tight curves.

4. CONCLUSIONS

Vehicles with forced-steered trucks adjust their geometry with track curvature to align their wheelsets in nominally radial positions. Because of the reduced wheelset misalignment in curves, a significant improvement in performance is possible by forced-steering the wheelsets in comparison to conventional suspensions. The results of this research show that understeering of the wheelsets can occur in vehicles with forced-steered trucks negotiating moderate and tight curves if kinematic principles alone are used to determine the steering linkage dimensions. Further, the results indicate that for forced-steered trucks with stiff steering linkage stiffnesses, steering gains slightly larger than the pure rolling line gain promise to offer performance improvements by decreasing (or eliminating) wheelset misalignments during moderate and tight curve negotiation.
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NOMENCLATURE

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<th>Symbol</th>
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<tr>
<td>b</td>
<td>half of truck wheelbase</td>
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<td>B</td>
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REFERENCES


